Scaffolding Process Based on Error Analysis in Solving the Problem: Second Order Differential Equation

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Abstract

Error analysis in solving mathematics problems is needed to be done so that the lecturer and the students can find out the students’ error in solving the problems. So, in the future the lecturer will administer the more appropriate scaffolding. The aim of this research is to analyze the appropriate scaffolding administered for the students based on error identification according to the Newman procedure of the students in solving second order differential equation. The method of this research was qualitative research case study design. The subject of the research was the students of mathematics education study program at IAIN Kudus who took differential equation class in the first semester in the academic year 2021/2022. The technique for data collection used were test and interview techniques. The result of the research of scaffolding administered for the students was by the techniques of inviting student participation, the students who performed comprehension errors obtained scaffolding in the form of offering explanations and verifying and clarifying student understandings, the students who performed transformation errors obtained offering explanations and modeling of desired behaviors techniques, the students who performed process skill errors obtained scaffolding in the form of Inviting students to contribute clues and Inviting student techniques, and the students who performed encoding errors obtained scaffolding in the form of Inviting student participation.

Keywords: Error Analysis; Mathematics; Scaffolding; Second Order Differential Equation

Abstrak

Analisis kesalahan dalam menyelesaikan masalah matematika perlu dilakukan agar dosen dan mahasiswa dapat mengetahui kesalahan mahasiswa dalam menyelesaikan masalah. Jadi, kedepannya dosen akan mengelola scaffolding yang lebih tepat. Penelitian ini bertujuan untuk menganalisis scaffolding yang tepat diberikan kepada siswa berdasarkan identifikasi kesalahan menurut prosedur Newman siswa dalam menyelesaikan persamaan diferensial orde dua. Metode penelitian ini adalah penelitian kualitatif dengan desain studi kasus. Subyek penelitian ini adalah mahasiswa program studi pendidikan matematika di IAIN...
Kudus yang mengambil mata kuliah persamaan diferensial pada semester I tahun ajaran 2021/2022. Teknik pengumpulan data yang digunakan adalah teknik tes dan wawancara. Hasil penelitian scaffolding yang diberikan kepada siswa adalah dengan teknik mengundang partisipasi siswa, siswa yang melakukan kesalahan pemahaman memperoleh scaffolding berupa menawarkan penjelasan dan memverifikasi dan memperjelas pemahaman siswa, siswa yang melakukan kesalahan transformasi memperoleh penjelasan menawarkan dan teknik pemodelan perilaku yang diinginkan, siswa yang melakukan kesalahan keterampilan proses memperoleh scaffolding berupa mengundang siswa untuk menyumbangkan petunjuk dan teknik Mengundang siswa, dan siswa yang melakukan kesalahan encoding memperoleh scaffolding berupa mengundang partisipasi siswa.

**Kata Kunci:** Analisis Kesalahan; Matematika; Scaffolding; Persamaan Diferensial Orde Dua

**Introduction**

One of the determinations of human sources is based on the education. Through education, the students can develop their skills to be faithful, moral, smart, creative, independent and become democratic and responsible citizens. One of the educations in Indonesia is through mathematics lesson (Noor, 2019; Wahyuni, Arthamevia, & Kurniawan, 2020; Ardiansyah, 2020; Richardo, 2020; Afifaturrohmaniyyah & Malasari, 2021; Taskiyah & Widyastuti, 2021; Riayah & Fakhriyana, 2021). An educator must encourage the students to work by their own thought in solving mathematics problems and give the chance to the students to develop their skills (Romberg, 2004). Mathematics learning cannot be separated from the ability to solve problems (Mulyono & Noor, 2017, p. 2). Mathematics provides the evidence of the truth of the idea and problem solving in logical, analytical, systematical, critical, and creative ways. Mathematics plays a significant role in the development of science, technology and information. Therefore, mathematics is considered as fundamental science and carried out in every level of education in Indonesia (Rosmaiayidi, 2018, pp. 59–70).

Differential equation class is available in the level of undergraduate mathematics education study program. In differential equation class, the students are required to explain, determine, and understand the definition and the concept of differential equation. The topics of the lecture include ordinary differential equation, linear differential equation, homogenous differential equation, first order differential equation and second differential equation. Differential equation presents a basic concept to solve mathematics models which can be found in science and engineering.

Mathematics learning in the university is influenced by the environment, the students and the lecturers as well as the use of the learning methods (Jana, 2018, pp. 1–7). If the students do not master mathematics, so they will perform mistakes in solving mathematics problems. Furthermore, the analysis of the error in solving
mathematics problems is needed to be done so that the lecturers are expected to find out the error as well as the reason of the error performed by the students in solving mathematics problems especially in this study regarding second order differential equation subject.

Related to the error in solving mathematics problems, it can be anticipated by studying and understanding the error further to prevent the same error in the future. Students’ errors in solving problems are varied (Muttaqi, Kartono, & Dwidayanti, 2021). The error in the first step caused the error in the next step, because the steps in solving mathematics problems are related to each other (Budi, Nusantara, Subanji, & Susiswo, 2020, p. 72). Analysis method used in this research is Newman error analysis method. Newman error analysis method was first time introduced in 1977 by Anne Newman, a mathematics teacher in Australia (Prakitipong & Nakamura, 2006, p. 113). In this method, Newman suggests five specific activities as a crucial step to guide the students in finding the error while solving word problems, they are: (1) reading, (2) comprehension, (3) transformation, (4) process skill, and (5) encoding.

Reading error occurred when the students read the words or the symbols in the questions incorrectly. A comprehension error occurred when the students are able to read the questions, but they failed to get what they need. Therefore, they are unsuccessful in solving a problem. A transformation error is an error which occurred when the students understand the questions well, but they are not able to choose the appropriate approach to solve the problems. A process skill error is an error which is performed by the students in the calculation process. An encoding error occurred when the students get the wrong answer to the question (Hafid, Kartono, & Suhito, 2016; Suyitno, 2018).

Furthermore, the information related to the mistakes in solving mathematics can be used by the lecturers to create a lesson plan or alternative learning method to decrease the possibility of the same error occurred in the future. The students are encouraged to think in their own ways, doing further investigation to explore their error, rather than to get the correct answer only (Boaler, 2000). Therefore, a lecturer is required to have an ability to understand and study the errors of their students further.

The lecturers need to optimize their role as a facilitator by making efforts to create a comfortable learning environment for the students. Learning innovation created to empowering the students need to consider the psychology and sociocultural aspect. One of the techniques represented the outlined mathematics learning pattern is scaffolding or mediated learning. The most frequent error made
by students in solving PISA 2012 questions based on error analysis according to Newman's procedure is encoding errors, the form of scaffolding given is asking students to understand the problem correctly so that they can make appropriate problem-solving strategies (Sari & Valentino, 2017, p. 90). The form of scaffolding that is carried out in solving absolute value questions after analyzing the errors according to Newman’s procedure is explaining, reviewing, restructuring and developing conceptual thinking (Budi et al., 2020, p. 69).

Scaffolding based on Vigotsky theory about socio-cultural cognitive development. Scaffolding as a learning method is a learning technique which allows the lecturer or the smarter students to mediate as well as adjusting the number of the supervised students based on the previous students’ achievements. Through scaffolding, the lecturer can change the level of guidance for the students. Therefore, the lecturer’s role as a learning facilitator can be optimized.

Scaffolding technique which will be used in this study is scaffolding technique based on Roehler and Cantlon (Bikmaz et al., 2010) there are 5 kinds of scaffolding techniques based on Roehler and Cantlon that are applied in learning mathematics; they are: (1) offering explanation; (2) inviting student participation; (3) verifying and clarifying student understanding; (4) modeling of desired behaviors and (5) inviting students to contribute clues. Offering explanation means the lecturer explains with explicit questions related to the material studied related to what, when, why and how to use it. Inviting student participation means giving chance to the students to be involved in the learning process. The lecturer gives an idea to solve the problem, while the students are encouraged to apply their knowledge and understanding. Verifying and clarifying student understanding, the lecturer verified the students’ response (understanding), whether their understanding is correct or not. If their understandings are incorrect, the lecturer gives clarification. Modeling of desired behaviors, the lecturer applies teaching technique to show how someone feel, think, or act in a certain situation. Inviting students to contribute clues, gives directions to the students to highlight the main concept of the given assignment. Those five techniques can be used together or independently depend on the material discussed (Bikmaz et al., 2010).

Based on the description above, it is important to study the process of scaffolding based on error analysis in solving second order differential equation. The aim of this study is to analyze the form of scaffolding (giving assistance) which is appropriate for the students based on the identification of the students’ error in solving the material of second order differential equation. The advantages of this research are: 1) for the researcher, can find out the error in solving second order differential equation and can give direct assistance to solve the problem, 2) for the
students, can find out their error and realizing the errors and correcting the errors, 3) for the educator, can get information about how to identify students’ error and how to solve it.

**Method**

The method used in this research was qualitative method. The aim of a qualitative method is to understand something experienced by the research subjects holistically and descriptive in the form of words by using scientific methods (Moleong, 2011, p.6). The aim of this qualitative research is to apply the appropriate scaffolding for the students based on their error analysis in solving second order differential equation. Case study design was chosen in this research to do the further research toward the research subjects. suggest that case study is a deep study about individual, group or institution (Lufri, 2007). The research subjects were the students of mathematics education study program at IAIN Kudus who took differential equation class in the first semester in the academic year 2021/2022. There were 3 classes with the total of 92 students.

The data collections in this research were test and interview techniques. The interview used is an unstructured interview related to how to solve the problem and explore the part of the error in answering the question. The researcher did direct communication with the respondents to find out the error in solving second order differential equation. The test applied in this research was written test in the form of 2-word problems of second order differential equation as an instrument with 50 minutes time allotment. The instrument had passed validity test; logical and empirical validity. According to (Arikunto, 2007, p.65) logic validity will be fulfilled if the instruments were arranged well according to the theory and determination. In this research, the instrument of logic validity had been arranged based on the material which would be evaluated then submitted to the expert validator who was a mathematics lecturer and the result showed that the 2 questioned were considered valid by validator I and validator II. Comments from validator I are the limits of the questions measured are clear, while validator II conveys that the questions are in accordance with the material being taught. The instrument which had been validated by the expert then was taken for empirical validation using the product moment correlation formula (Arikunto, 2007, p.72) through a try out instrument from 29 respondents.

The result of try out instrument of test material of second order differential equation was reliable with question number 1 was valid intermediate difficulty and accepted power difference, while question number 2 was valid with advance difficulty and accepted power difference. The test used to find out the error which
had been done by the students in solving second order differential equation problems which further analyzed using Newman procedure.

Data analysis technique used in this research refers to (Sugiyono, 2012) they are data collection, data reduction, data presentation, and conclusion. The first data was data collection and correction of the students answers of the test of second order differential equation problems. The step of data reduction in this research was analyzing the error of the students’ answers by using Newman procedure and transformed in a note as materials for the interview. The step of data presentation in this research was presenting the error in each problem by Newman procedure and presenting recorded interview results. In the conclusion step, by comparing analysis of the students’ work on the test with the interview result to find out the students’ error in solving second order differential equation problems and determining the appropriate scaffolding to reduce the possibility of the same error in the future.

Results

Based on the result of the analysis of the students’ answer on the test of the second order differential equation problems, different errors made by the students occurred. The errors were analyzed based Newman procedure; they are reading errors, comprehension errors, transformation errors, process skill errors, and encoding errors. If the errors found were the same, therefore only one error was discussed. Research subjects M1, M2, M3, M4, and M5 had errors on the test items consisting of 2 questions. Research subjects M1, M2, M3, M4, and M5 had errors on the items consisting of 2 questions. The selection of subjects M1, M2, M3, M4, and M5 is based on the consideration that they have different faults and are communicative when spoken to so that they can easily find answers. The error analysis of research subjects in solving second-order differential equations, in terms of the type of error based on the Newman procedure, is presented in Table 1 below.
Table 1. Error analysis of research subjects in terms of the type of error based on the Newman procedure

<table>
<thead>
<tr>
<th>Research Subjects</th>
<th>Question Number</th>
<th>Error Type</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>R</td>
</tr>
<tr>
<td>M1</td>
<td>1</td>
<td>v</td>
</tr>
<tr>
<td></td>
<td>2</td>
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<tr>
<td>M2</td>
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<tr>
<td></td>
<td>2</td>
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<tr>
<td>M3</td>
<td>1</td>
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<tr>
<td></td>
<td>2</td>
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<tr>
<td>M4</td>
<td>1</td>
<td>-</td>
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<tr>
<td></td>
<td>2</td>
<td>-</td>
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<tr>
<td>M5</td>
<td>1</td>
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</tr>
</tbody>
</table>

Description:
R: reading errors
C: comprehension errors
T: transformation errors
P: process skill errors
E: encoding errors
v: there is an error
x: did not answer or there was an error due to a previous error
-: no error

Table 1 shows that a few subjects made reading errors, comprehension errors, and encoding errors, while the most common errors were process skill errors. After analyzing the students’ error using Newman procedure, then the errors were analyzed in the form of the appropriate scaffolding to avoid the possibility of the same error occurs in the future. The following presents 5 problem solving of the students who made different errors-based Newman procedure and their scaffolding.

Discussion

After analyzing errors according to Newman’s procedure, students were given scaffolding. The purpose of providing scaffolding is to build student knowledge so that they do not make mistakes again when faced with other questions. There are many techniques in providing scaffolding. In this research, using scaffolding technique from Roehler and Cantlon (Bikmaz et al., 2010) they are: (1) offering explanation; (2) inviting student participation; (3) verifying and
clarifying student understanding; (4) modeling of desired behaviors and (5) inviting students to contribute clues. The five techniques can be used simultaneously or separately depending on the material to be discussed (Kurniasih, 2012, p. 113). The choice of scaffolding technique from Roehler and Cantlon in this study was based on the fact that the technique does not only focus on the teacher in providing scaffolding but also involves students to play an active and responsive role in learning by providing opportunities for students to contribute ideas (Cahyono & Pribady, 2020, p. 59).

The analysis of the student’s error in solving second order differential equation will be discussed further in each error occurred according to Newman procedure. The problem number 1 is chosen to discuss the detail of the error because the errors according to in problem number 1 Newman procedure occurred in problem number 1. The problem number 1 is differential equation: \( y'' - 3y + 2y = 4x^2 + 3x + 2 \), determine the solution which meets initial condition \( y(0) = 1 \) dan \( y'(0) = -1 \).

The first error according to Newman procedure is reading errors. The example of reading errors can be seen in Figure 1 as follow.

Figure 1. The example of reading errors

Based on the answer from student M1 in Figure 1 can be found out that student M1 read the question incorrectly; homogenous differential equation in the problem was \( y'' - 3y + 2y = 0 \) but the student solved the problem with equation: \( y'' + 3y' + 2y = 0 \). The error of student M1 in reading the problem caused incorrect answer. Moreover, it was incorrect from the characteristic root. Although the steps in solving the problem were correct, but because it was incorrect from the beginning, therefore incorrect result occurred. To confirm the data, the researcher
conducted interview with student M1. M1 confess inaccuracy in reading the problem because he/she was in hurry to solve the problem.

Scaffolding which was given to student M1 who did reading errors in problem number 1 was Inviting student participation technique; that is by giving clues and directions to reread correctly and accurately so the same error will not occur in the future.

The second error was comprehension errors. The example of comprehension errors can be seen in Figure 2 as follow.

Figure 2. The example of comprehension errors

```latex
Fungsi tebak.
\begin{align*}
y &= At^2 + Bt + C + De^t \\
y' &= 2At^2 + B + De^t \\
y'' &= 4At^2 + De^t 
\end{align*}
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Figure 2 showed that student M2 made error in understanding the problem; that was determining a correct guess function according to problem number 1. The correct guess function was: \( y_p = Ax^2 + Bx + C \), therefore when it was substituted to its differential equation, it did not show the correct result. In the interview with student M2, he/she confess that he/she has not really understood how to determine a correct guess function.

Scaffolding given to student M2 who did comprehension errors in problem number 1 was Offering explanations technique; that was by explaining how to solve differential equation using guess function then continued Verifying and clarifying student understandings technique; that was by giving questions and directions to reflect and correct the answer written, so student M2 realized the error and understood the whole steps in determining the correct problem solving in differential equation.

Based on Newman procedure, the third error was transformation errors. The example of transformation errors can be seen in Figure 3 as follow.
The problem solving of student M3 for problem number 1 in Figure 3 showed that student M3 made an error in transforming characteristic root to determine characteristic function. Characteristic function which was determined from the characteristic roots were $\lambda_1 = 2$ and $\lambda_2 = 1$ the correct characteristic roots were $y_1 = e^{2t}$ and $y_2 = e^t$. In the problem solving of student M3, he/she wrote the correct characteristic roots but the characteristic functions were incorrect. Therefore, in determining the solution for homogenous differential equation was incorrect. In the interview with student M3, he/she said that he/she did not realize that it was incorrect.

Scaffolding given to student M3 who did transformation errors in problem number 1 was Offering explanations technique; that was by explaining how to determine characteristic function from characteristic roots and by Modeling of desired behavior technique; that was by giving example of the use of characteristic function and asked student M3 to reflect and recall the previous knowledge, therefore he/she realized the error and was able to solve the problem correctly.

The fourth mistake according to Newman was process skill errors. The example of process skill errors can be seen in Figure 4 as follow.
Scaffolding Process Based on Error Analysis in Solving the Problem...

Process skill errors in Figure 4 based on the problem solving of student M4 was miscalculation in calculating Wronskian value $W(y_1, y_2)$ the correct value was $e^{3x} - 2e^{3x} = -e^{3x}$. Therefore, in determining the solution, the inhomogeneous equation was also incorrect. Interview was conducted to find out the cause of the error. The result showed that student M4 told that he/she did the problem solving carelessly. Scaffolding given to student M4 so he/she will not make the same error in the future by using Inviting students to contribute clues technique; that was by giving questions to help him/her realizing the mistake and asked student M4 to be more careful in solving the problems.

The fifth error based on Newman procedure was encoding errors. The example of encoding errors in solving second order differential equation in this research can be seen in Figure 5 as follow.
Student M5 made an error in writing the final answer, whereas the steps and the calculations were correct. The value of $C_2 = -10$ was correct, but in writing the solution of this problem, he/she wrote $+10$. It can be seen from the previous answer that he/she erased, but he/she did not revise it. In the interview, student M5 confesses that he/she was in hurry to solve the next problem and did not recheck. Scaffolding given was Inviting student participation technique; that was by asking the student to recheck the answer before submitted, so the error will not occur.

There are several challenges faced by lecturers in providing scaffolding, namely the number of students, the language of instruction and time constraints (Kamil, 2018, p. 187). The limitation of this study is that the effectiveness of the designed scaffolding has not been tested. This becomes a recommendation for further research. The scaffolding provided by the teacher involves student participation, provides clear direction and reduces student confusion (Noviansyah, 2015, p. 947). In principle, the scaffolding is given and then the scaffolding is reduced and finally eliminated after the students really gain an understanding (Kurniasih, 2012, p. 113).

**Conclusion**

Based on the research result, it can be concluded that few subjects made reading errors, comprehension errors, and encoding errors, while the most frequent errors were process skill errors. Scaffolding given to the students with reading errors was inviting student participation technique; that is by giving clues and directions to reread correctly and accurately so the same error will not occur in the future. The students who did comprehension errors received scaffolding with offering explanations techniques; that was by explaining the material and Verifying
and clarifying student understandings; that was by asking the students to reflect and correct the answer. Scaffolding given to the students with transformation errors was offering explanations technique; that was by explaining the material and Modeling of desired behaviors technique; that was by giving the example of the problem solving with the same type of problem. The student with process skill errors received scaffolding by inviting students to contribute clues technique; that was by giving questions so the students realized the mistake. Students with encoding errors received scaffolding by inviting student participation technique; that was by asking them to recheck the answer before submitted.

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