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Designing Probability Learning Utilizing Ice Cream Sticks: An RME-Based Approach

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Abstract

This study aims to design a learning trajectory for probability using the context of ice cream sticks through the Realistic Mathematics Education (RME) approach. The method employed is design research of the validation study type, consisting of preliminary design, teaching experiment, and retrospective analysis. Data were collected through student activity sheets and interviews involving 31 ninth-grade students. The results indicate that the designed learning trajectory facilitates a gradual shift in students' reasoning from informal to formal probability concepts. At the initial stage, students identified possible outcomes through concrete activities involving colored sticks. They then constructed sample spaces by listing outcomes and representing them using tables and tree diagrams in two-stage experiments. At the model-for stage, students were able to determine the total number of possible outcomes and distinguish between types of events, although some difficulties were observed in understanding sampling with replacement. At the formal stage, most students expressed probability as a ratio of favorable outcomes to the total sample space and represented it using symbolic notation. Retrospective analysis shows that the Hypothetical Learning Trajectory (HLT) is valid in supporting students' conceptual development, particularly in bridging contextual understanding and formal representation. However, teacher guidance remains necessary in constructing complete sample spaces and interpreting probability as a ratio. This study contributes a practical context-based learning trajectory for implementing RME in probability learning.

Keywords: Ice Cream Sticks; Probability; RME

Abstrak

Penelitian ini bertujuan untuk mendesain lintasan belajar (learning trajectory) pada materi peluang menggunakan konteks stik es krim melalui pendekatan Realistic Mathematics Education (RME). Metode yang digunakan adalah design research tipe validation study yang terdiri dari tahap preliminary design, teaching experiment, dan retrospective analysis. Data dikumpulkan melalui lembar aktivitas siswa dan wawancara dengan melibatkan 31 siswa kelas IX. Hasil penelitian menunjukkan bahwa lintasan belajar yang dirancang mampu mendukung pergeseran penalaran siswa secara bertahap dari konsep peluang informal menuju formal. Pada tahap awal, siswa mengidentifikasi kemungkinan hasil melalui aktivitas konkret mengenali warna stik. Selanjutnya, siswa mengonstruksi ruang sampel dengan menuliskan seluruh kemungkinan dan merepresentasikannya dalam bentuk tabel serta diagram pohon pada percobaan dua tahap. Pada tahap model-for, siswa mulai mampu menentukan banyaknya seluruh kemungkinan kejadian dan membedakan jenis-jenis kejadian, meskipun masih ditemukan kesulitan dalam memahami konsep pengambilan dengan pengembalian. Pada tahap formal, sebagian besar siswa mampu menyatakan peluang sebagai perbandingan antara banyaknya kejadian yang diinginkan dengan seluruh kemungkinan serta menuliskannya dalam bentuk notasi simbolik. Hasil retrospective analysis menunjukkan bahwa Hypothetical Learning Trajectory (HLT) yang dikembangkan valid dalam mendukung perkembangan pemahaman konsep siswa, khususnya dalam menjembatani transisi dari penalaran kontekstual menuju representasi matematis formal. Namun demikian, intervensi guru melalui pertanyaan penuntun masih diperlukan, terutama dalam membantu siswa menyusun ruang sampel secara lengkap dan memahami peluang sebagai suatu perbandingan. Penelitian ini menghasilkan desain lintasan belajar materi peluang berbasis konteks yang praktis dan dapat diterapkan dalam pembelajaran berbasis RME di kelas.

Kata Kunci: Es Krim Stik; Peluang; RME

Introduction

Mathematics plays a central role in developing students' reasoning and problem-solving abilities needed to address real-world situations (Lafuente-Lechuga et al., 2020). Accordingly, mathematics education is expected to foster logical, analytical, and critical thinking skills rather than merely emphasizing procedural mastery (Arisoy & Aybek, 2021), enabling students to address real-life problems (Istikomah et al., 2024). Nevertheless, many students still perceive mathematics as difficult because mathematical concepts are often presented in abstract forms that are disconnected from meaningful contexts (Al-saaidi et al., 2025).

Despite its relevance to daily life, several mathematical topics of mathematics are still difficult for students to understand in a meaningful way. One persistent challenge lies in learning probability. Although closely related to everyday phenomena such as games and random events (Wijaya et al., 2021), students often struggle to understand it meaningfully. Many rely on memorizing formulas without grasping underlying concepts, resulting in errors in identifying sample spaces and determining probabilities (Andam et al., 2025; Sari et al., 2023). This issue is exacerbated by instructional practices that emphasize procedural calculation over conceptual understanding and provide limited opportunities for students to explore randomness through contextual experiences (Cholily et al., 2025; Khairi et al., 2025).

To address these challenges, a meaningful and context-based approach is required. Realistic Mathematics Education (RME) offers a relevant framework, emphasizing that mathematical knowledge should be constructed by students through engagement with meaningful contexts (Trisnawati et al., 2026). Rooted in Freudenthal's perspective, RME views mathematics as a human activity, where learning begins with phenomena that are experientially real to students (didactical phenomenology) (Freudenthal, 1973). Through guided reinvention, students are supported in rediscovering mathematical ideas by progressing from informal strategies toward formal concepts (Siswantari et al., 2025). This process is often represented in the Iceberg model, where visible informal reasoning forms the foundation for deeper formal mathematical understanding (Sari et al., 2025). Such principles are particularly important in probability learning, where students need concrete experiences before engaging with symbolic representations.

A simple yet powerful context that supports this process is the use of ice cream sticks. As familiar and easily accessible materials, they enable students to engage in hands-on activities such as drawing lots, grouping, and recording outcomes. These activities allow students to experience randomness directly, explore possible outcomes, and gradually construct sample spaces and probability concepts as ratios. In this way, ice cream sticks function not merely as contextual tools but as mediators that bridge informal experiences and formal mathematical reasoning, aligning with the core principles of RME.

Previous studies have demonstrated that contextual and concrete activities in RME support students' understanding of probability concepts. For instance, game-based contexts such as snakes and ladders (Aisy et al., 2024) and physical activities (Ahadiya et al., 2024) help students explore sample spaces and empirical data. However, these approaches often rely on complex or resource-intensive settings that may not be practical for everyday classroom implementation. Moreover, limited attention has been given to how simple, low-cost contexts can be systematically designed to support students' transition from informal to formal reasoning within RME framework.

Therefore, this study proposes a learning design using ice cream sticks as a simple, flexible, and accessible medium. Compared to previous studies, this approach offers a more practical alternative while maintaining strong pedagogical value. The design is grounded in the principles of Realistic Mathematics Education (RME), particularly Freudenthal's view that mathematics should be experienced as a human activity through meaningful contexts and guided reinvention (Freudenthal, 1973). Through hands-on exploration with ice cream sticks, students are encouraged to develop informal reasoning about random events before gradually moving toward formal probability concepts, reflecting the Iceberg model in which abstract mathematical understanding emerges from contextual experiences (Rahmi et al., 2025). The novelty lies in utilizing a minimalist context that still supports structured exploration and conceptual development of probability in a meaningful and accessible way.

Based on this rationale, the study aims to design a learning trajectory for probability using the context of ice cream sticks within the RME framework. The study contributes an empirically grounded trajectory that demonstrates how a simple contextual medium can support students in progressing from informal reasoning toward formal probability understanding.

Method

The research subjects were 31 grade IX students in the 2025/2026 academic year from one junior high school in Palembang, Indonesia. The class was selected purposively based on the teacher's recommendation and the suitability of the probability topic in the learning schedule, enabling the implementation of the designed activities. This study employed design research with a validation study type, which is widely used to develop and refine instructional designs in real classroom settings (Handayani et al., 2025). This approach was chosen because the aim of this study was not only to implement learning, but also to systematically design, test, and improve a learning trajectory (Rengganis & Hidayat, 2025) for probability based on students' learning processes. Design research allows researchers to explore how students construct their understanding through iterative cycles, making it particularly suitable for studies grounded in the Realistic Mathematics Education (RME) approach (Wisnu et al., 2025), which emphasizes learning through meaningful contexts and progressive formalization. The research consisted of three main stages: preliminary design, design experiment, and retrospective analysis (Ramadhan et al., 2022). The first stage, preliminary design, involves developing a Hypothetical Learning Trajectory (HLT), which includes learning objectives, contextual activities, and predictions of students' thinking as a foundation for the learning design (Lestari & Marsigit, 2020).

The second stage is the design experiment, which consists of two phases: the pilot experiment and the teaching experiment (Anggriani et al., 2025). Both phases are integral parts of the experimental design aimed at testing and refining the proposed learning trajectory. The pilot experiment was conducted with 6 students divided into two small groups to examine the feasibility of the learning design and to revise activities that were not yet appropriate. The teaching experiment was then carried out with the whole class to investigate the implementation of the refined design and to observe students' actual learning trajectory (ALT) in the context of probability learning using ice cream sticks. The data in this study were obtained from classroom interactions, student responses, group discussions, student activity sheets, and interviews, which served as the main data sources for analysis.

The final stage is retrospective analysis, which involves analyzing the data from the design experiment and comparing the Hypothetical Learning Trajectory (HLT) with the Actual Learning Trajectory (ALT) (Apriasiska et al., 2025). The ALT refers to the real learning pathway experienced by students during the teaching experiment, which is then compared with the predicted pathway in the HLT to examine the suitability of the design for students' learning development (Risnanosanti et al., 2023; Sanita et al., 2024).

Data were collected through student worksheets and semi-structured interviews. The data analysis technique was conducted qualitatively through systematic description and narration of the data obtained from LAS and interviews. The analysis process involved organizing the data, identifying patterns in students' responses, and describing students' thinking processes, strategies, and difficulties that emerged during the learning activities (Siregar et al., 2024). The constructed narrative illustrates the development of students' understanding from the informal stage to the formal stage in accordance with the learning trajectory in the RME approach. To ensure the validity of the data, triangulation was carried out by comparing the results from the LAS and interview data. In addition, peer discussion and validation with the teacher were conducted to ensure the consistency and credibility of the findings. The flow of the research stages can be seen in the following chart.

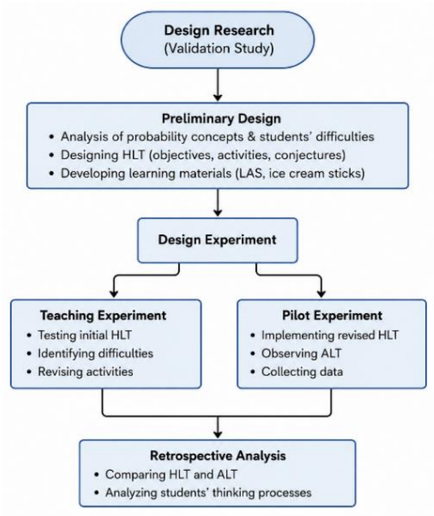


Figure 1. Design Research Process

Results

Preliminary Design

The preliminary design revealed that students' main difficulty in learning probability lies in their reliance on formal formulas without a prior intuitive understanding of randomness and possible outcomes. Most students were unable to identify the sample space completely and tended to focus only on observed outcomes rather than considering all possible events. This indicates that students have not yet developed a foundational understanding of probability as a set of possibilities. The use of colored ice cream sticks as a learning context provided a concrete representation that supported students in recognizing randomness and exploring possible outcomes. Through simple activities such as randomly drawing sticks and identifying their colors, students began to realize that all outcomes must be considered, not only those that appear. This suggests that a tangible and familiar context plays a crucial role in bridging students' initial intuitive thinking toward a more structured understanding of probability. Furthermore, the selection of ice cream sticks was not only based on practicality but also on their potential to facilitate repeated random experiments in a controlled yet flexible manner. This allows students to observe patterns, compare outcomes, and gradually construct the idea of sample space before transitioning to formal representations. Therefore, the preliminary design indicates that learning grounded in concrete and accessible contexts can effectively address students' conceptual gaps in probability, particularly in understanding randomness and completeness of possible outcome.

The learning design was developed by integrating the main characteristics of RME into a sequence of classroom activities (Pratiwi & Meiliasari, 2025). The learning process began with the use of a real context, where students engaged in randomly picking ice cream sticks to introduce the concept of probability in a concrete manner. This activity was then followed by the use of models as a bridge to formal understanding, as students organized their experimental results into tables that represent the possible outcomes. Interactivity is emphasized through small group discussions, allowing students to compare their findings and construct shared understanding. Furthermore, the principle of intertwinement is incorporated by encouraging students to connect the results of their experiments with related mathematical concepts, such as ratios and fractions, thereby supporting a deeper and more integrated understanding of probability.

The following HLT (Hypothetical Learning Trajectory) contains three main components, namely objectives, activity descriptions, and conjectures.

Table 1. Hypothetical Learning Trajectory (HLT)

Stage	Activity	Objective	Activity Description	Conjecture
Informal (con-tex-tual)	Take 1 colored ice cream stick from the contain-ner at random	Students can understand that the results of random experiments cannot be predicted with certainty and mention the possible colors that may appear	The teacher prepares a container with 5 ice cream sticks of different colors. Students take 1 stick at random and announce the result. The teacher then asks: “If it's not this color, what other colors might come out?”	a. Students only mention the colors that appear. b. Some students begin to mention all colors. c. Some students realize that all colors have an equal chance.
Model of	Write down all possible out-comes of the stick draw	Students identify the space of possibilities (the beginning of the sample space) and begin to present it in the form of a list/picture	After the discussion, the students write down all the possible colors that could come out (yellow, red, pink, green, and cream). The teacher confirms that these are all the possible outcomes.	a. Students write incomplete answers (e.g., only their favorite color). b. Students begin to realize that the list must include all colors. c. Some students draw sticks

				according to color.
Model for	Create a table or tree diagram for drawing 2 sticks (with or without replace-ment)	Students are able to represent and determine the possible outcomes of drawing 2 sticks (with or without replacement) using tables or tree diagrams	The teacher asks students to develop 2 sticks. Students make a list of pairs (M-B, B-M, M-H, etc.) or use a table/tree diagram.	a. Students are confused about distinguishing sequences. b. Students try to write manuals but miss pairs. c. Some students realize they need a system (table or tree).
Formal	Determi ning probabil ity using the formula $P(A) = \frac{n(A)}{n(S)}$	Students are able to calculate the probability of a simple event using the concepts of sample space and events	Students are guided to constru-ct the probability formula by compa-ring successful outcomes with the total possible out-comes from repea-ted ice cream stick draws. Through discussion, stude-nts recognize probability as the ratio of desired events to total possibilities. For example, they con-clude that the probability of sele-cting a green stick was $\frac{1}{5}$, while the probability of obtaining two sticks of the same color with repla-cement was $\frac{5}{25} = \frac{1}{5}$. The teacher then formalise stude-nts' reasoning into the conventional probability formula.	a. Students initially calculate directly using estimates ("it seems like 1 to 5"). b. After looking at the table/diagram, students understand the idea of n(A)/n(S). c. Students can generalize to other problems.

The HLT was then analyzed and mapped using the Iceberg model in the RME approach to describe the students' thinking process from the informal to the formal stage. Through the Iceberg model, context-based learning of probability using ice cream sticks began with concrete and contextual experiences (informal level), then developed through situational and referential models (intermediate level), and finally led to symbolic and formal understanding (formal level) of the concept of probability. Before implementation, the researchers discussed with teachers to review the suitability of the design to classroom conditions. Teachers assessed that the use of ice cream sticks would attract students' interest and be easy to implement because it did not require special tools. Teachers also suggested that the activity carried out in small groups to facilitate observation and increase interaction among students.

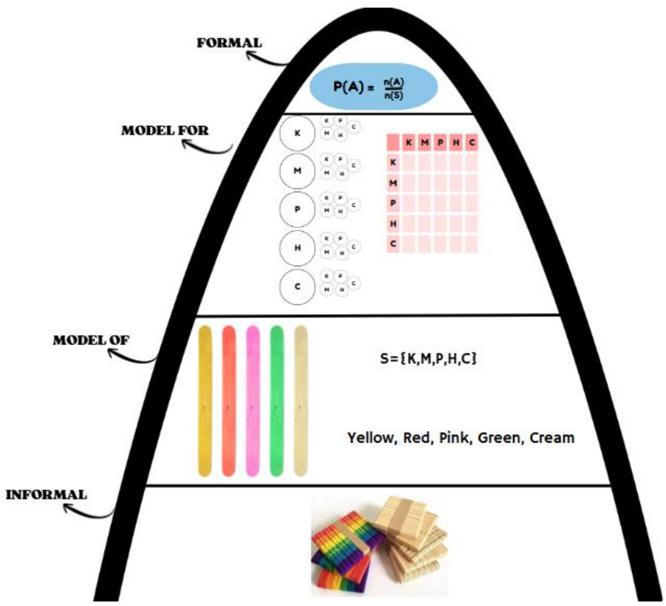


Figure 2. Iceberg

Design Experiment

The design experiment stage was the phase of implementing the learning design using the ice cream stick context developed in the preliminary design stage. The main objective of this stage is to test, analyze, and refine the HLT through iterative teaching and learning processes in real classroom settings. The design experiment consists of two stages, namely the pilot experiment and the teaching experiment (Irma & Nada, 2024).

Pilot Experiment

The pilot experiment was conducted after the completion of the HLT design in the preliminary design phase. This stage aimed to examine the feasibility of the probability learning design using the context of ice cream sticks before its implementation in the teaching experiment stage. The participants consisted of six ninth-grade students who were purposively selected to represent different levels of mathematical ability (high, medium, and low). The students were divided into two groups to enable researchers to observe variations in students' strategies, responses, and difficulties during the learning process. The teacher and researchers acted as facilitators who guided discussions and documented students' thinking throughout the activities.

The lesson began by introducing a container filled with yellow, red, pink, green, and cream-colored ice cream sticks. The HLT conjecture tested in this activity was that students would develop an intuitive understanding of probability through recognizing uncertainty and equal opportunity in random events. When students were asked to randomly select one stick, they showed enthusiasm and began discussing whether each color had the same chance of being selected. Most students concluded that all colors had equal opportunities to appear, indicating that the conjecture was achieved. This activity showed that the contextual situation encourages students to express informal reasoning before using formal probability concepts.

In the next activity, students were asked to construct tables and tree diagrams representing all possible outcomes from selecting two sticks. The HLT conjecture predicted that students would be able to organize all possible outcomes systematically using these representations. The conjecture partially held. Students were generally successful in constructing tables, indicating that they could identify combinations of outcomes correctly. However, several students experienced difficulties in completing the tree diagrams systematically, particularly in maintaining the sequential structure of outcomes. The students' work also revealed that although they understood the idea of the sample space, they still had difficulty distinguishing between sample space and sample points. Based on these findings, the HLT was revised by adding guiding questions and scaffolded examples of incomplete tree diagrams in the worksheet to support students in organizing sequential outcomes more systematically.

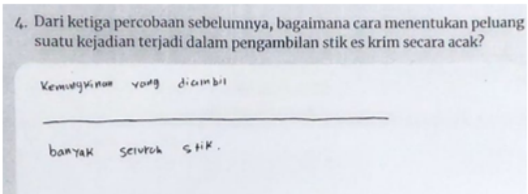
 4. Dari ketiga percobaan sebelumnya, bagaimana cara menentukan peluang suatu kejadian terjadi dalam pengambilan stik es krim secara acak? Kemungkinan yang diambil / banyak seluruh stik.	**English version:** 4. Based on the three previous experiments, how do you determine the probability of an event occurring when randomly selecting an ice cream stick? $\frac{probability\ of\ selection}{total\ number\ of\ sticks}$

Figure 3. Analyzing Opportunities in Symbolic Terms

Figure 3 presents students' responses when determining the probability of an event. The HLT conjecture in this activity was that students would express probability as a comparison between the number of desired outcomes and the total number of possible outcomes. Students were able to explain probability informally using statements such as "the possibility of being taken" and "a lot of the whole stick," which indicated an understanding of probability as a ratio between favorable outcomes and total outcomes. However, students tended to express their reasoning verbally rather than symbolically and did not consistently use formal mathematical notation such as fractions. This finding indicates that students had developed an intuitive understanding of probability but still required support in transitioning to formal symbolic representation. Therefore, the HLT was revised by adding questions that explicitly prompted students to represent their reasoning using mathematical symbols and fraction notation.

The results of the pilot experiment suggest that the ice cream stick context is feasible for supporting students' transition from informal reasoning toward formal probability concepts. The pilot experiment also identifies several aspects of the HLT that require refinement, particularly in supporting students' understanding of sample points, tree-diagram representations, and symbolic expressions of probability before broader implementation in the teaching experiment stage.

Teaching Experiment

The teaching experiment revealed how students developed their understanding of probability through the ice cream stick context. Through two main RME-based activities, students actively engaged in identifying possible outcomes, constructing sample spaces, and representing results using tables and tree diagrams. These activities enabled students to move from informal reasoning toward more structured representations, showing gradual conceptual development. While the learning was conducted in one session with 31 students working in groups, the primary findings highlighted that the designed activities effectively supported students in recognizing patterns of possible outcomes and beginning to interpret probability as a ratio.

Activity 1

Question 1: Identifying the sample space (S) of an experiment

In question 1, students were asked to identify the sample space (S) of an experiment involving randomly drawing one ice cream stick, where each color has an equal probability. Students' responses can be grouped into three types of representation: visual, verbal, and symbolic.

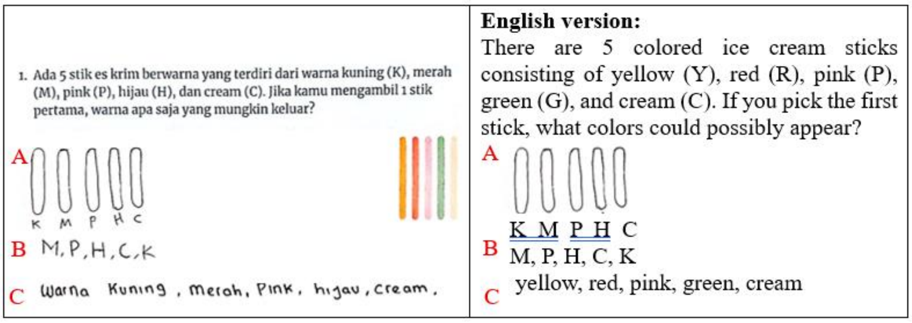


Figure 4. Ability to Identify Possible Colors That Will Appear

In the visual representation, students drew five colored sticks and labeled them (K, M, P, H, C), indicating their ability to connect the concrete context with symbols. In the verbal representation, students listed the colors in words (yellow, red, pink, green, cream), showing their understanding of possible outcomes. In the symbolic representation, students wrote the sample space using letters (M, P, H, C, K), although the order varied, the answers remained correct since order does not affect the sample space. Overall, these responses reflected a progression from informal to formal representation in line with the RME learning trajectory.

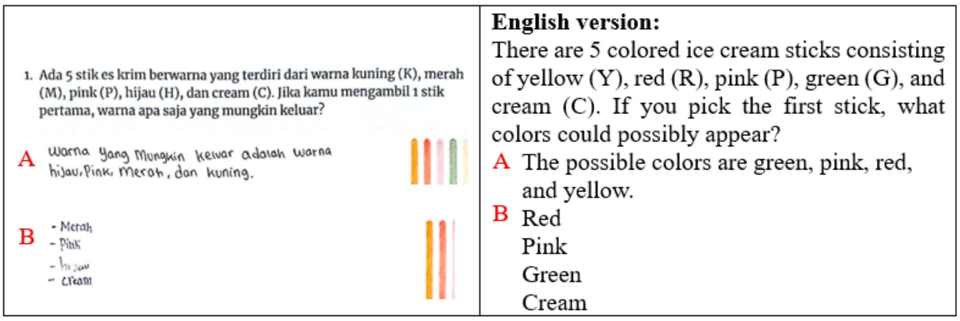
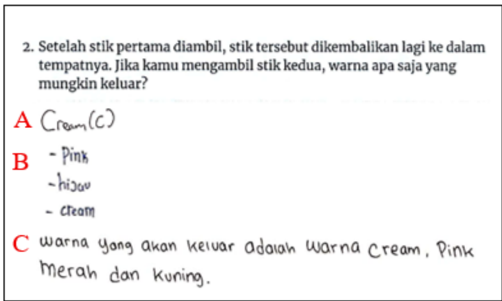


Figure 5. Errors in Identifying Possible Colors That May Appear

However, two groups did not include one color, namely cream and yellow, resulting in an incomplete sample space. This finding indicates that although students had begun to grasp the basic idea of probability, they had not yet fully understood the necessity of listing all possible outcomes in a sample space. This error can be classified as an incomplete sample space misconception, where students fail to consider all elements of the outcome set. In addition, it also reflects a partial conceptual understanding, as students focus only on observable or remembered outcomes rather than systematically identifying all possibilities. The activity using ice cream sticks has successfully fostered students' conceptual understanding gradually. Most students were able to identify the possible colors that might appear, demonstrating an initial understanding of the concept of sample space, even though some misconceptions were still evident in their responses.

Question 2: Understanding the concept of sampling with replacement

The second question tested students' understanding of the concept of sampling with replacement, namely the probability on the second draw remained the same as on the first draw, because the number and type of sticks returned to their original state. However, the results shown in Figure 8 reveal several errors. In Figure 8(a), the student only wrote down one color. This indicates that the student did not understand that the question referred to all possible outcomes (sample space), rather than a single outcome.


2. Setelah stik pertama diambil, stik tersebut dikembalikan lagi ke dalam tempatnya. Jika kamu mengambil stik kedua, warna apa saja yang mungkin keluar?

A Cream (C)

B - Pink
- hijau
- cream

C warna yang akan keluar adalah warna cream, pink merah dan kuning.

English version:
After the first stick is taken out, it is put back into its place. If you take out the second stick, what colors might come out?

A Cream (C)

B -Pink
-Green
-Cream

C The colors that will come out are cream, pink, red, and yellow.

Figure 6. Determine The Colors that Might Appear in The Second Draw

In Figure 8 (C), students did not write down the color green, indicating that they did not understand that all colors have the same probability after the sticks are returned. In Figure 8 (B), students only wrote down three colors out of the five available colors. This shows that students still thought that colors that had been taken would not appear again. Students thought concretely and contextually, associating the physical process (taking sticks) with a reduction in the number of sticks, as can be seen from the following dialogue.

Teacher : Please explain why in number 2, you only wrote down three colors, namely pink, green, and cream? Aren't there five colors on the stick?

Student : The thing is, in number 1, we already took one stick, so there are only four sticks left. Now, in number 2, we took another one, so there are only three colors left that haven't come out yet.

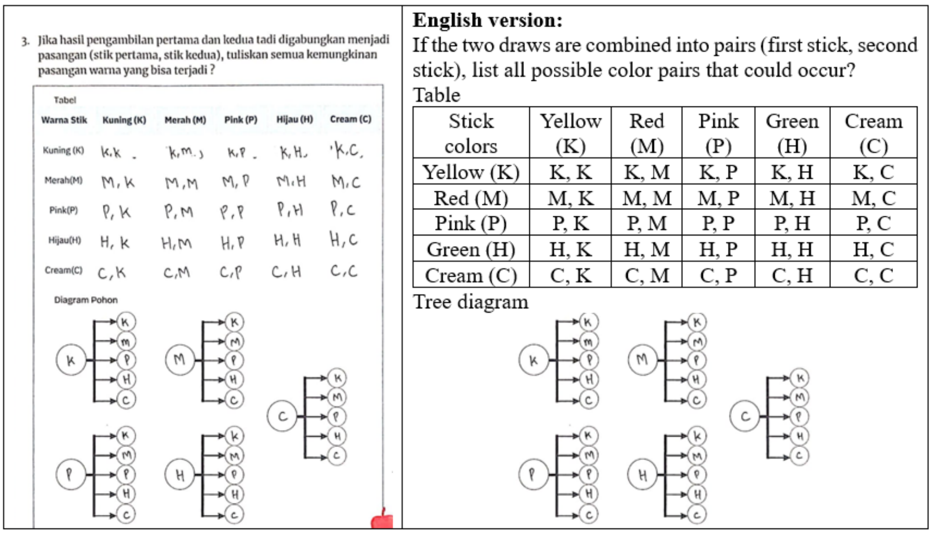
Teacher : Oh, I see. So, according to you, the stick that was taken earlier was not put back?

Student : Yes, we don't think it will be returned, ma'am. So, what has been taken cannot be returned.

This dialogue shows that students still misunderstood the idea of replacement in probability. They interpreted the activity concretely, assuming that sticks that had already been taken could not appear again. As a result, students formed an incomplete sample space because they believed each draw reduced the possible outcomes of the next draw. The teacher's probing questions helped students rethink their assumptions without directly answering, reflecting the RME principle of guided reinvention. Through this interaction, students gradually realized that all colors remained possible outcomes after each stick was returned.

Question 3: Determining the sample space of two events

This question asked students to determine the sample space of two events through the concept of multiplying events or forming sequential pairs in the context of ice cream sticks.



Stick colors	Yellow (K)	Red (M)	Pink (P)	Green (H)	Cream (C)
Yellow (K)	K, K	K, M	K, P	K, H	K, C
Red (M)	M, K	M, M	M, P	M, H	M, C
Pink (P)	P, K	P, M	P, P	P, H	P, C
Green (H)	H, K	H, M	H, P	H, H	H, C
Cream (C)	C, K	C, M	C, P	C, H	C, C

Figure 7. Group 3 Answers: Tables and Tree Diagrams

The students' answers are presented in the form of a table and a tree diagram. In the table representation, students listed all possible ordered pairs from the first and second stick selections, such as K,K, K,M, and so on, until all possible combinations were completed. Meanwhile, in the tree diagram representation, students illustrated branching possibilities from each color to all other colors, showing the sequential process of the first and second selections. These two representations indicate that students began to understand that each color in the first selection can be paired with all available colors in the second selection because the sticks are returned after each draw.

Through this activity, students were expected to reach the formal stage of the HLT by understanding all possible pairs and representing them systematically in the form of a table or tree diagram. However, in question number 3, group 1 initially experienced difficulties, as seen in the following dialogue:

Student	:	Ma'am, what do you mean by color pairings? We're confused, do you mean the same color or what?
Teacher	:	That's a good question. Try to remember, when we took the first and second sticks, could the colors be the same or different?
Student	:	It can vary, ma'am. Sometimes yellow comes out first, then pink. But it can also be the same.
Teacher	:	Well, that's right. Now imagine that you pick a yellow stick first. What colors might come out for the second stick?
Student	:	Hmm... it could be red, pink, green, cream, or yellow again, ma'am.
Teacher	:	Great. So, if the first stick is yellow, what kind of pair can you make?
Student	:	Meaning (K, K), (K, M), (K, P), (K, H), and (K, C).
Teacher	:	Yes, great! Now, what if the first stick is red?
Student	:	That means (M, K), (M, M), (M, P), (M, H), and (M, C), Ma'am.
Teacher	:	Amazing! Now you see the pattern: each first color can be paired with all second colors. You can continue with all colors and write them down in the table.

From the dialogue above, it can be seen that the students were confused about making tables and tree diagrams, but after being given leading questions from the teacher, they were finally able to make tables and tree diagrams.

Question 4: Understanding the meaning of a single sample point

This question aimed to test the students' understanding of the meaning of a single sample point from the sample space they had formed. All groups answered correctly. Question number 4 was, "*From the list of color pairs you ' ve written down, give one example of a possible outcome of the experiment involving the selection of those two sticks!".* Based on students' worksheets, their answers varied according to the pairs in the table or tree diagram, showing that the students understood the concept of a single sample point as a single result from the sample space they had formed.

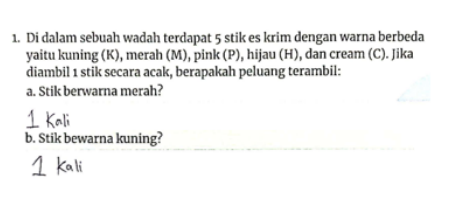
Question 5: Counting possible pairs

Question number 5 was, "*How many possible pairs are there in total?*". All groups answered that there were 25 possible color pairs. This result is in accordance with the compound sample space of 5 ice cream stick colors taken twice with replacement.

Activity 2

Question 1: Probability of an event

Activity 2 aimed to help students discover the concept of simple probability through the context of ice cream sticks, so they understand that the probability of an event is the ratio between the number of desired events and all possible outcomes. In question number 1, five groups were able to conclude that each color had an equal chance of being picked, namely one in five.


1. Di dalam sebuah wadah terdapat 5 stik es krim dengan warna berbeda yaitu kuning (K), merah (M), pink (P), hijau (H), dan cream (C). Jika diambil 1 stik secara acak, berapakah peluang terambil:
a. Stik berwarna merah?
1 Kali
b. Stik bewarna kuning?
1 Kali

English version:
There are 5 ice cream bars of different colors in a container yellow (K), red (R), pink (P), green (H), and cream (C). If one bar is selected at random, what is the probability of selecting:

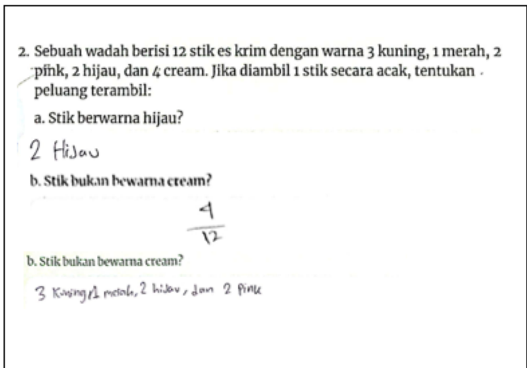
a. A red bar?
   1 time
b. A yellow bar?
   1 time

Figure 8. Group 6's Mistake in Answering Question 1 of Activity 2

This understanding did not arise immediately, but rather through interactive dialogue between the teacher and students. The teacher prompted the students with guiding questions such as: "How many sticks are there in total?" and "How many sticks are red?". However, one group was still at the concrete thinking stage, where they only counted the number of sticks of a certain color without relating it to the total number of sticks (n(S) = 5), so they did not yet understand probability as a ratio.

Question 2: Probability of two or more events

On question number 2, most groups answered correctly. In part 2a, one group made a mistake by only writing "*2 green*," which shows that they understood the number of green sticks (n(A)), but did not represent it as a probability by comparing it to the total number of sticks (n(S) = 12).


2. Sebuah wadah berisi 12 stik es krim dengan warna 3 kuning, 1 merah, 2 pink, 2 hijau, dan 4 cream. Jika diambil 1 stik secara acak, tentukan peluang terambil:
a. Stik berwarna hijau?
2 Hijau
b. Stik bukan bewarna cream?
$\frac{4}{12}$
b. Stik bukan bewarna cream?
3 Kuning, 1 merah, 2 hijau, dan 2 pink

English version:
2. A container holds 12 ice cream bars: 3 yellow, 1 red, 2 pink, 2 green, and 4 cream-colored. If one bar is selected at random, determine the probability of selecting:

a. A green bar?
   2 Green
b. A bar that is not cream-colored?
   $\frac{4}{12}$
b. A bar that is not cream-colored?
   3 yellow, 1 red, 2 green, and 2 pink

Figure 9. Incorrect Answer to Question 2, Activity 2

In part 2b, two groups made mistakes. The first group wrote down the probability $\frac{4}{12}$, which shows a misunderstanding of the number of non-cream sticks; the number 4 is actually the number of cream sticks. The second group wrote down a list of colors: "*3 yellow, 1 red, 2 green, and 2 pink*," which shows that they understood the meaning of not cream, but were unable to convert that information into a formal probability.

Question 3: Concept of compound probability with replacement

This question aimed to help students understand the concept of compound probability with replacement, where each draw has an equal chance. At first, students seemed confused about determining the number of identical and different pairs. However, after the teacher guided them with leading questions and reminded them to look back at the sample space table from the previous activity, their understanding improved.

c. Terambil stik berwarna merah kemudian stik berwarna pink.	**English version:** C. Take the red stick, then the pink stick
merah, kuning Pink, kuning	red, yellow pink, yellow
merah, merah Pink, merah	red, red pink, red
merah, pink Pink, Hijau	red, pink pink, green
merah, hijau Pink, pink	red, green pink, pink
merah, cream Pink, cream	red, cream pink, cream

Figure 10. Group 2's Incorrect Answer to Question 3C

In Figure 10, the students' mistake is apparent when they wrote down all pairs of colors that start with red and pink. This shows that they could construct a composite sample space, but did not yet understand that the probability requested was for the specific event of "red followed by pink," not all pairs containing the colors red and pink.

Teacher	:	Please explain why you only wrote down colors that start with red and pink?
Student	:	Because you asked about red and pink, ma'am. So, we wrote down combinations that have red and pink.
Teacher	:	Okay, so according to you, the first color can only be red and pink, is that right?
Student	:	Yes, because the question is red and then pink.

This result indicates that students experienced a misconception in interpreting ordered events in compound probability. Although they were able to construct the composite sample space, they did not yet understand that the event requested was specifically "red followed by pink," rather than all combinations containing red and pink. Students focused only on the keywords in the question without considering the sequence of events. Besides that, the interview confirms that students interpreted the problem superficially. They selected all pairs involving red and pink because they assumed the question referred to any combination of those colors. This shows that students had not yet distinguished between unordered combinations and ordered outcomes in probability situations.

Question 4: The concept of probability is a ratio.

Question 4 was designed to help students develop an understanding of probability as a ratio between the number of desired events and the total number of possible outcomes. Through this task, students were expected to move beyond simply listing outcomes and begin expressing probability in a more formal mathematical form. The following student response illustrates how the student interpreted probability by relating favorable outcomes to all possible outcomes in the experiment.

4. Dari ketiga percobaan sebelumnya, bagaimana cara menentukan peluang suatu kejadian terjadi dalam pengambilan stik es krim secara acak? $\frac{\text{Jumlah warna stik yang di minta}}{\text{Jumlah seluruh warna stik}}$	**English version:** Based on the three previous trials, how do you determine the probability of a certain outcome when randomly selecting an ice cream stick? $\frac{number\ of\ sticks\ requested}{total\ number\ of\ stick\ colors}$

Figure 11. Students' Understanding from Informal Representation to Formal Representation

The student's answer above shows that the student understood that probability is a ratio. The student mentioned the correct components, namely the numerator, which is the number of desired events (the requested colors), and the denominator, which is the total number of possible outcomes (all stick colors). This shows that the student could generalize (draw general patterns from several examples), which is the goal of the formal stage in RME.

Question 5: Formal stage of the concept

In question number 5, students successfully continued their understanding by writing it down in mathematical symbols, showing that they had reached the formal stage completely.

5. Nyatakan dalam bentuk matematika cara menentukan peluang suatu kejadian dari jawaban nomor 4? $P(A) = \frac{n(A)}{n(S)}$	**English version:** Express mathematically how to determine the probability of an event based on the answer to question 4? $P(A) = \frac{n(A)}{n(S)}$

Figure 12. Formal Stage of The Concept of Probability

This achievement did not happen suddenly, but as the result of the teacher asked a guiding question, "How do you write the number of desired events in mathematical symbols?" Then, the teacher linked the symbols A and S to events and sample spaces. The teacher then provided a simple example on the board, such as: A = a specific event and S = all possibilities. With these events, the students were encouraged to write the formula themselves, rather than copying it. In this way, the students built their own understanding from the teacher, in accordance with the RME principle that the teacher is a facilitator.

Retrospective Analysis

In the retrospective analysis stage, the researchers compared the Hypothetical Learning Trajectory (HLT) with the Actual Learning Trajectory (ALT) that emerged during the pilot experiment and teaching experiment (Apriasiska et al., 2025). The analysis was based on students' worksheets, interviews, and group discussions. Overall, the ALT generally followed the conjectures designed in the HLT, although several revisions were needed during implementation.

At the initial stage, students were able to recognize the sample space through the activity of randomly selecting ice cream sticks. Most students identified the possible outcomes correctly, although some students did not list all available colors. This finding indicates that the contextual activity has helped students develop an initial understanding of uncertainty and possible outcomes before using formal probability concepts.

In the model development stage (model of and model for), students represented possible outcomes using tables and tree diagrams. Most students were successful in constructing tables systematically, but several students experienced difficulties in developing complete tree diagrams, particularly in situations involving sampling with replacement. Some students assumed that a selected stick would not reappear in the next draw. Through guiding questions and classroom discussions, students gradually recognized that the stick was returned before the next selection, allowing them to revise their representations and identify all possible outcomes correctly. Based on these findings, revisions to the HLT were made by adding scaffolded tree-diagram tasks and guiding questions to support students in organizing sequential outcomes systematically.

At the formal stage, most students were able to conclude that probability is determined by comparing the number of desired outcomes with the total number of possible outcomes. Students also began to express probability using symbolic notation such as n(A) and n(S). However, some students initially expressed their reasoning verbally rather than symbolically. Therefore, additional prompts encouraging symbolic representation were added to the worksheet to support students in transitioning from informal explanations to formal mathematical notation.

The results of the retrospective analysis suggest that the ice cream stick context has supported students' transition from informal reasoning toward formal probability concepts. Nevertheless, difficulties remained in understanding sampling with replacement and constructing tree diagrams systematically, indicating the need for further refinement of the learning trajectory before broader classroom implementation.

Discussion

The findings indicate that the ice cream stick context has supported students in developing an intuitive understanding of probability through concrete experiences. Students were able to recognize uncertainty and identify possible outcomes before using formal symbolic representations. This finding is consistent with previous studies showing that contextual activities in RME learning help students bridge informal reasoning and formal mathematical concepts (Mariadi et al., 2025).

The use of tables and tree diagrams also revealed that students developed different levels of understanding during the mathematization process. Most students were able to organize possible outcomes using tables, but difficulties emerged when constructing tree diagrams, especially in sampling with replacement situations. Similar findings have been reported in previous probability-learning studies, which found that students often struggle to represent sequential events systematically and tend to assume that previously selected objects cannot reappear (Zega et al., 2025).

In addition, the gradual transition from verbal reasoning to symbolic notation reflects the principle of progressive mathematization in RME and PMRI (Asmaarobiyah et al., 2025). Students initially explained probability using everyday language before expressing it mathematically using symbols such as n(A) and n(S). The teacher's guiding questions and classroom discussions played an important role in supporting this transition from contextual understanding to formal abstraction (Ahmad, 2026).

Overall, the study suggests that the ice cream stick context can support students' conceptual development in probability learning, particularly in helping students move from concrete experiences toward formal mathematical representations. However, the findings also indicate that additional scaffolding is still needed to strengthen students' understanding of sampling with replacement and tree-diagram representations.

Conclusion

This study demonstrates that a learning trajectory for probability based on the RME approach using ice cream sticks effectively supports students' conceptual development from informal to formal reasoning. A key finding of this study is that the use of a simple and low-cost context, such as colored ice cream sticks, can systematically facilitate students in constructing complete sample spaces, understanding sequential events, and interpreting probability as a ratio through progressive modeling activities (from model-of to model-for). Unlike previous studies that rely on complex or large-scale contexts, this study shows that a minimalist context can still generate rich mathematical thinking when supported by well-designed tasks and teacher guidance. In addition, the findings highlight the crucial role of guided questioning in helping students overcome misconceptions, particularly in understanding sampling with replacement and organizing outcomes using tables or tree diagrams. Therefore, this study contributes a practical and scalable learning design that can be easily implemented in classroom settings while still maintaining the core principles of RME. Future studies are encouraged to extend this approach to more complex probability topics or explore other simple contexts to enrich RME-based learning designs.

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