



Designing a GeoGebra-Integrated Learning Trajectory for Fractions Using *Jombang's* Traditional Land Measurement

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Abstract

The low level of understanding of fractions among students indicates the need for more contextual, meaningful, and relevant learning in everyday life. This study aims to develop a Hypothetical Learning Trajectory (HLT) based on Realistic Mathematics Education (RME) integrated with GeoGebra. The HLT uses the context of traditional land measurement in Jombang to support students' understanding of fractions. This study employs a design research method consisting of three main stages: preliminary design, design experiment, and retrospective analysis. It focuses on the preliminary design stage and the pilot experiment phase, involving six seventh-grade students selected through purposive sampling. The instruments used included HLT-based Student Worksheets, observation sheets, and interview guidelines. Data were collected through the implementation of HLT in the pilot experiment stage, observation of student activities during the completion of Student Worksheets, and interviews after the activity. Data were analyzed descriptively and qualitatively. The results showed that the developed HLT was able to facilitate students' transition from informal to formal understanding, as indicated by an improvement in the quality of their conceptual understanding in connecting the context, visual representations, and symbolic notation of fractions. GeoGebra was found to help students visualize fraction concepts more concretely, while the local context made learning more relatable and meaningful. Theoretically, these findings reinforce the RME principles of guided reinvention and phenomenological exploration within a learning trajectory. This HLT can serve as a reference for designing interactive, contextual, and culturally aligned fraction learning experiences that align with students' experiences and cultural backgrounds.

Keywords: Design Research; Ethno-RME; Fractions; GeoGebra; HLT

Abstrak

Rendahnya tingkat pemahaman siswa terhadap pecahan menunjukkan perlunya pembelajaran yang lebih kontekstual, bermakna, dan relevan dalam kehidupan sehari-hari. Penelitian ini bertujuan untuk mengembangkan Hypothetical Learning Trajectory (HLT) berbasis Realistic Mathematics Education (RME) yang

diintegrasikan dengan GeoGebra. HLT ini menggunakan konteks pengukuran lahan tradisional di Jombang untuk mendukung pemahaman siswa terhadap pecahan. Penelitian ini menggunakan metode *design research* yang terdiri dari tiga tahap utama: *preliminary design*, *design experiment*, dan *retrospective analysis*. Fokus penelitian ini adalah pada tahap *preliminary design* dan fase *pilot experiment*, yang melibatkan enam siswa kelas tujuh yang dipilih melalui *purposive sampling*. Instrumen yang digunakan meliputi Lembar Kerja Peserta Didik (LKPD) berbasis HLT, lembar observasi, dan pedoman wawancara. Data dikumpulkan melalui penerapan HLT pada tahap *pilot experiment*, observasi aktivitas siswa selama pengerjaan LKS, serta wawancara setelah kegiatan. Data dianalisis secara deskriptif dan kualitatif. Hasil penelitian menunjukkan bahwa HLT yang dikembangkan mampu memfasilitasi siswa dalam transisi dari pemahaman informal ke formal, yang ditunjukkan oleh peningkatan kualitas pemahaman konseptual mereka dalam menghubungkan konteks, representasi visual, dan notasi simbolik pecahan. GeoGebra terbukti membantu siswa memvisualisasikan konsep pecahan secara lebih konkret, sementara konteks lokal membuat pembelajaran lebih relevan dan bermakna. Secara teoretis, temuan ini memperkuat prinsip RME mengenai *guided reinvention* dan *phenomenological exploration* dalam sebuah *learning trajectory*. HLT ini dapat menjadi referensi dalam merancang pengalaman pembelajaran pecahan yang interaktif, kontekstual, dan sesuai budaya, yang selaras dengan pengalaman serta latar budaya siswa.

Kata Kunci: Design Research; Ethno-RME; GeoGebra; HLT; Pecahan

Introduction

Fractions are a fundamental mathematical concept that plays an important role in meeting human needs and simplifying various aspects of life (Švecová, Balgová, & Pavlovičová, 2022). In everyday life, fractions frequently appear in various situations, such as sharing food, measuring cooking ingredients, and calculating land areas. Mastery of fractions is an important foundation for understanding advanced mathematical material (Irwan & Murti, 2023). Therefore, a good understanding of fractions not only supports mathematical literacy but also contributes to meaningful learning that is relevant to everyday life (Anriana et al., 2024).

Although fractions have been taught since elementary school, they remain a challenging topic for students. A national survey shows that around 30% of junior high school students still struggle with basic fraction operations. Additionally, research indicates that students can typically solve procedural problems but fail to grasp the underlying conceptual meaning (Lazuardi et al., 2017). The conventional and abstract teaching approach also exacerbates this situation, making mathematics learning feel boring and difficult for students to access (Irwan & Murti, 2023). As a result, students tend to have difficulty understanding concepts, show low interest,

and struggle to relate knowledge to real-life situations. In fact, learning often ends up being merely memorizing formulas (Yanrizawati et al., 2023).

To overcome students' difficulties in understanding fractions, learning needs to be designed to be more meaningful and contextual. One approach that can be used is Realistic Mathematics Education (RME). RME was developed by Freudenthal with the principle that mathematics is a human activity. According to Hans, the key principles of RME include guided reinvention, phenomenological didactics, and self-developed models. The RME approach uses real contexts as the starting point for learning. This allows students to relate abstract concepts to their experiences.

An important strategy for implementing RME involves developing Hypothetical Learning Trajectories (HLTs). An HLT is a conjectured model of the learning process, comprising learning goals, a sequence of instructional activities, and hypotheses about the progression of students' thinking (Kamphorst, Vollebregt, Savelsbergh, & van Joolingen, 2023). The design of an effective HLT is guided by specific design principles that ensure a coherent transition from informal to formal understanding, such as the use of meaningful contexts and structured reinvention of concepts (Lerman, 2020). This principle has been operationalized in the Indonesian context, for example, in designing a learning trajectory for set theory using local shadow puppet stories as culturally meaningful contexts (Risdiyanti & Prahmana, 2021)

According to the epistemological foundation of RME, mathematical knowledge is a human activity that must be developed and reinvented by students through guided interactions with situations from everyday life, rather than a fixed body of truth to be transmitted (Lerman, 2020). By integrating local culture and technology in real-life situations that spur mathematical reasoning, this study reinforces this theoretical stance. For students, learning fractions is a natural and significant human activity because of the setting of traditional land measurement in Jombang, which offers a culturally and historically accurate starting point. Additionally, the use of GeoGebra as a dynamic modeling tool operationalizes the RME principles of self-developed models and guided reinvention in the digital age by extending students' intuitive, context-based understanding into formal mathematical representations. (Ziatdinov & Valles, 2022).

Research (Yulia et al., 2020) shows that HLT for fractions based on RME, designed through a design research approach, can facilitate students in reconstructing fraction concepts gradually, from concrete contexts to formal understanding. In its development, the HLT can also be enriched through the integration of local cultural elements through an ethnomathematical approach. The

combination of real-life contexts and local cultural values, known as Ethno-RME, not only enhances conceptual understanding but also fosters students' appreciation of their cultural heritage.

One potential local context is the traditional land measurement system in Jombang District, which employs local units such as the bahu. This system is widely used in land transactions and involves operations involving fractions and unit conversions, which are relevant for learning. Unfortunately, this potential has not been widely utilized in formal education.

Several previous studies have explored the application of RME and the development of HLT in various mathematics topics using diverse local contexts. For example, RME-based HLT has been designed for topics such as three-dimensional shapes, using contexts such as the Central Java Grand Mosque (Fitriyana & Nursyahidah, 2022), the Megono Gunung tradition in cone material (Nursyahidah et al., 2021), historical buildings like the Sam Poo Kong Temple in reflection and translation material (Lestari et al., 2021), and traditional seasonal systems such as Pranata Mangsa in statistics learning (Nursyahidah et al., 2024). However, none of these studies has integrated technology, local cultural contexts, and learning trajectory design simultaneously.

The integration of dynamic technology, such as GeoGebra, in HLT development can further strengthen the guided reinvention process in the RME approach. GeoGebra is software that enables the visualization of mathematical concepts. The use of software such as GeoGebra can help students explore fraction concepts with more concrete visual representations, thereby improving their understanding and engagement in learning (Rosyidi et al., 2024). A study conducted by T. A. Putri et al. (2025) also shows that the integration of GeoGebra in e-worksheets with local cultural content significantly improves students' conceptual understanding and makes learning more effective and contextual. Research by Rosyidi et al. (2024) also demonstrates that interactive learning environments using GeoGebra can effectively support students' mathematical problem-solving processes, highlighting the potential of digital tools to enhance engagement and understanding in Indonesian classrooms.

The epistemological contribution is that it holistically embodies three components: the structured pedagogical framework inherent in RME-based HLT, the genuine and richly cultural setting found in traditional land measurement practices in Jombang, and the dynamic modeling capacity for GeoGebra. Through this integration, the study aims to create a powerful learning environment in which

students' progression from informal, culturally grounded understanding to formal mathematical reasoning is systematically guided and visually supported.

Based on this background and identified gap, the research question in this study is how a GeoGebra-assisted RME-based Hypothetical Learning Trajectory (HLT), designed using the context of Jombang's traditional land measurement system, supports students' conceptual understanding of fractions. This study aims to develop an RME-based HLT design that integrates the use of GeoGebra and the context of traditional land measurement in Jombang for fraction learning. This study will also examine the practicality and effectiveness of the design in supporting students' understanding of fraction concepts. Theoretically, this study is expected to expand the study of HLT development through the integration of technology and local culture. Practically, the results of this study can be used as a guide for teachers in designing more contextual, meaningful, and relevant fraction learning for students.

Method

This study employed a design-based research approach within a developmental research framework. The design research method was chosen as the research approach because this study aims to produce an integrated GeoGebra-based HLT design and test its effectiveness in helping students understand fractions in the context of traditional land measurement in Jombang. This method enables researchers to iteratively design, test, and refine instructional materials, making the outcomes more responsive to students' learning needs and cultural contexts (Nursyahidah et al., 2025). It is characterized by its interventionist, theory-generative, and iterative nature (Prediger et al., 2015). Various studies have demonstrated that design research is effective in improving the quality of mathematics learning by constructing Local Instruction Theories (LIT) based on evidence from the classroom (Adha et al., 2024).

The design research method comprises three stages: preliminary design, design experiment, and retrospective analysis (Aripin et al., 2025). This study focused on the preliminary design stage and the pilot experiment phase of the design experiment, which served as the initial testing of the HLT. The teaching experiment, involving implementation in a larger classroom, was conducted in the next phase of the research.

In the preliminary design stage, the researcher reviewed literature on fraction concepts, the context of traditional land measurement in Jombang, and the research methods used. The literature review served as the basis for designing the

LIT, which consists of learning objectives and student hypotheses (Firmansari, Herman, & Nurlaelah, 2024). In the design experiment stage, the HLT was tested through two phases: a pilot experiment and a teaching experiment (Astuti & Wijaya, 2021). The pilot experiment was conducted on a small group of students with varying abilities to collect initial data and revise the HLT. The teaching experiment was then implemented in a regular classroom setting to assess the feasibility and effectiveness of the revised design (Nursyahidah et al., 2025). The retrospective analysis stage aimed to reflect on and analyze the learning process based on the collected data and to compared it with the designed HLT to determine its suitability.

The conceptual framework that supported this research consisted of Hypothetical Learning Trajectory (HLT), Realistic Mathematics Education (RME), and GeoGebra integration. The HLT was designed based on student hypotheses, consistent with the principal idea that effective learning trajectories must be grounded in learners' intuitive understanding (Adha et al., 2024). The RME approach was used to create contextual and meaningful learning by linking fraction material to the context of traditional land measurement in Jombang. The integration of GeoGebra as a tool for visualization and mathematical exploration is expected to facilitate students' concrete understanding of fraction concepts, as emphasized by Nursyahidah et al. (2024).

The participants in this study were six seventh-grade junior high school students aged 12–13 years, consisting of two female students and four male students who were deliberately selected for their high mathematical ability. This deliberate choice was made because the pilot experiment aimed to evaluate the clarity, challenge level, and potential of the HLT with students capable of engaging in higher-level mathematical reasoning, prior to broader classroom implementation. Participants were selected through purposive sampling, which is commonly used in qualitative studies to select individuals based on characteristics that are most relevant to the research purpose (Subhaktiyasa, 2024). The inclusion criteria were: (1) being in seventh grade, (2) possessing high mathematical ability, and (3) expressing willingness to participate in the entire research process. The exclusion criteria included students who were absent during the data collection process or did not complete all stages of the research. The small number of participants was chosen because this study was a pilot experiment, which aimed to collect initial data to improve learning design before the teaching experiment (Utari et al., 2024).

The research instruments included student worksheets (LKPD), observation sheets, and interview protocols. Throughout the learning sessions, the LKPD was used to record the students' thought processes. Students' responses and behaviors

during the HLT's deployment, especially during the pilot phase, were recorded using observational data collection methods. Interviews were conducted as well to learn more about the students' experiences and intellectual grasp of fractions.

An expert review verified the validity of the research tools, such as the LKPD, observation sheets, and interview protocols. This evaluation was carried out by an expert lecturer with expertise in design research, RME, and mathematics education. The evaluation concentrated on construct validity, instrument clarity, and content validity, including conformity with learning objectives and RME principles. The instruments were finalized by carefully incorporating the expert's recommendations. Both qualitative and descriptive methodologies were used to analyze the data. Thorough data triangulation, which included cross-referencing student workbooks, observational recordings, and interview transcripts, increased the findings' validity. This procedure offered a thorough and comprehensive assessment of how students' fraction knowledge changed as the HLT was implemented.

The LKPD was administered, the HLT integrated with GeoGebra was implemented, in-class observations were conducted, and post-activity interviews were conducted as part of the data-gathering process. The collected data were analyzed using descriptive and qualitative methods to examine how students engaged with the HLT and how their understanding of fractions evolved during the learning process, in line with previous studies on RME-based HLT development (Yulia et al., 2020).

Results

The learning process in this study was organized using the iceberg model framework, which facilitated students' development of abstract mathematical concepts in real-world contexts. This layered evolution progresses from contextual activities to generalization and formalization, as depicted by the iceberg model in Figure 1. An HLT was created based on this model to help students move through these phases in an organized and significant way.

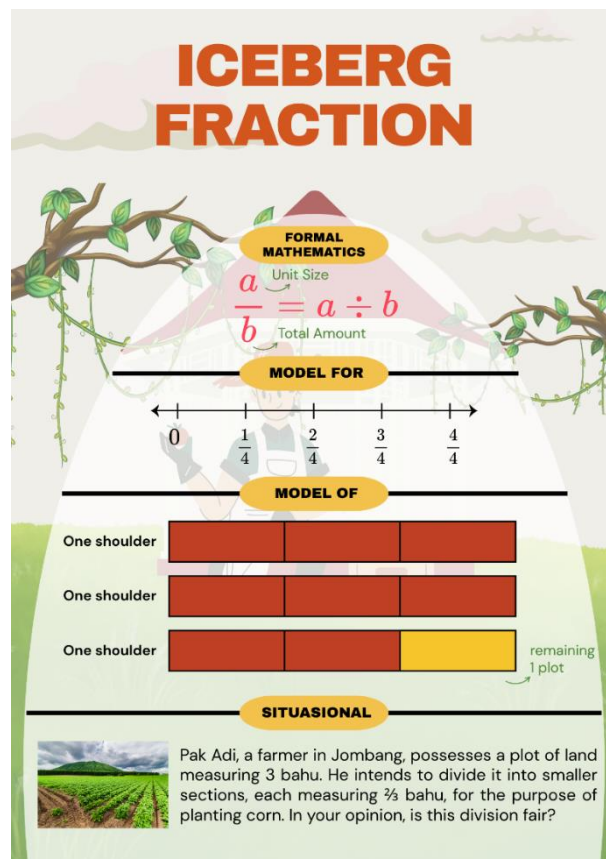


Figure 1. Iceberg Model of Fraction

In alignment with this framework, several learning activities were designed based on the hypothetical learning trajectory and students' cognitive processes related to the concept of fractions. The fraction learning trajectory was structured into four learning activities, each of which aimed to build students' understanding gradually from informal contextual experiences to formal mathematical representations. These activities were developed based on the principles of Realistic Mathematics Education (RME) and integrated with the local cultural context, including traditional land units in Jombang, along with dynamic visualizations using GeoGebra. The series of learning activities in the HLT consisted of four steps, described as follows.

Activity 1: Contextual understanding through storytelling

This activity aims to activate students' informal understanding of fractions through a contextual scenario involving land division using a local non-standard unit (bahu). Students were presented with a narrative about Pak Adi, who owned three *bahu* of land and planned to divide it into plots of two-thirds *bahu*. They were

asked to reflect on fairness in division, the implications of using local measurement units, and how Pak Adi could ensure accuracy in dividing the land.

Table 1. Comparison between HLT & ALT of activity 1

Learning Activity	HLT	ALT
Reflecting on the importance of fair division	<ol style="list-style-type: none"> 1. Students state that fair division is important to ensure everyone receives the same amount and to avoid disputes. 2. Students may not yet consider fairness important, focusing only on task completion. 	<ol style="list-style-type: none"> 1. Students explain that fair division is important to avoid harming others. 2. Students mention that equal division helps ensure successful land use or farming.
Identifying challenges of using traditional units like <i>bahu</i>	<ol style="list-style-type: none"> 1. Students recognize that <i>bahu</i> is not standardized and can lead to confusion across regions. 2. Students do not consider unit standardization as a problem due to local familiarity. 	<ol style="list-style-type: none"> 1. Students explain that <i>bahu</i> is not a standard unit and may cause misunderstanding. 2. Students suggest using standard units such as hectares for broader clarity.
Suggesting a way to divide 3 <i>bahu</i> into $\frac{2}{3}$ portions	<ol style="list-style-type: none"> 1. Students divide each <i>bahu</i> into 3 equal parts and take 2 parts from each. 2. Students misunderstand $\frac{2}{3}$ as simply dividing 3 by 2 or confuse it with sharing 2 <i>bahu</i> among 3 people. 	<ol style="list-style-type: none"> 1. Students divide each <i>bahu</i> into 3 parts and choose 2 parts from each. 2. Students mention using a fraction formula to determine how many $\frac{2}{3}$ plots fit into 3 <i>bahu</i>.

Table 1 presents a comparison between the Hypothetical Learning Trajectory (HLT) and the Actual Learning Trajectory (ALT) from Activity 1. The ALT data show that most students were able to explain the importance of fair division and recognize that *bahu* is not a standardized unit, showing that they could connect contextual fairness with mathematical equality. This indicates that the use of storytelling about local land division effectively elicited students' initial reasoning

about fractions through familiar experiences. A key finding is that the contextual problem successfully functioned as a phenomenological exploration. It situates the fraction concept within a realistic and culturally relevant scenario, which is the first principle of RME.

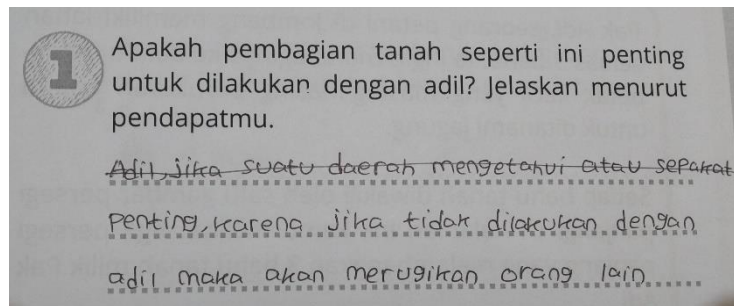


Figure 2. Student's written response to a contextual question in Activity 1

Figure 2 illustrates one student's written response to the initial question about fairness in dividing land. The student stated that fair division is important because if it is not done fairly, it could harm others. This response shows early reasoning rooted in real-life experience, which aligns with the RME principle of starting from familiar situations.

Activity 2: Visual representation and reasoning

The activity aims to help students represent fractional division visually by modeling the problem of dividing three *bahu* of land into plots of two-thirds *bahu* each. Students are asked to draw and divide shapes to represent each *bahu*, determine how many full plots of $\frac{2}{3}$ can be made, and identify any leftover land. This activity develops students' part-whole understanding and reasoning through repeated grouping.

Table 2. Comparison between HLT & ALT of activity 2

Learning Activity	HLT	ALT
Drawing and dividing 3 rectangles to represent land areas	<ol style="list-style-type: none"> Students divide each rectangle into 3 equal parts and shade 2 parts in each to represent $\frac{2}{3}$. Students may draw each shoulder as one rectangle and divide it into two parts instead of three. 	<ol style="list-style-type: none"> Student divides 1 shoulder into 2 parts, thinking that taking $\frac{2}{3}$ of 3 shoulders means dividing 1 shoulder into 2. Student interprets $\frac{2}{3}$ as 2 plus 3, then divides 1 shoulder into 5 parts. Student correctly divides 1 shoulder into 3 equal parts.

Table 2. Comparison between HLT & ALT of activity 2

Learning Activity	HLT	ALT
Calculating the number of $\frac{2}{3}$ plots and identifying leftover land	1. Students determine that 4 plots of $\frac{2}{3}$ can fit into 3 <i>bahu</i> with $\frac{1}{3}$ <i>bahu</i> left.	1. Students correctly state that 4 plots of $\frac{2}{3}$ <i>bahu</i> can be made from 3 <i>bahu</i> , with $\frac{1}{3}$ <i>bahu</i> left. 2. Students say that 6 plots of $\frac{2}{3}$ can be made, but also mention that $\frac{1}{3}$ <i>bahu</i> remains.
Determining strategies to divide 3 <i>bahu</i> into plots of $\frac{2}{3}$	1. Students represent the division using visual models such as rectangles or repeated subtraction. 2. Students identify that multiple strategies can be used to divide 3 <i>bahu</i> into equal plots of $\frac{2}{3}$.	1. Students state that there is only one way to divide the land. 2. Students state that there are other possible strategies besides the one used in the task.

Table 2 presents a comparison between the Hypothetical Learning Trajectory (HLT) and the Actual Learning Trajectory (ALT) from Activity 2. The findings show how students used visual strategies, including estimation and partitioning, to analyze and depict two-thirds of a *bahu*. The visual scaffolding and task instructions were improved in light of these variations to better assist students' reasoning when relating part-whole relationships with contextual measurement. The ALT reveals a process of guided reinvention, in which teachers intentionally use students' diverse strategies, whether they are initially correct or not, to assist the creation of more formal models of division that are consistent with the core ideas of RME.

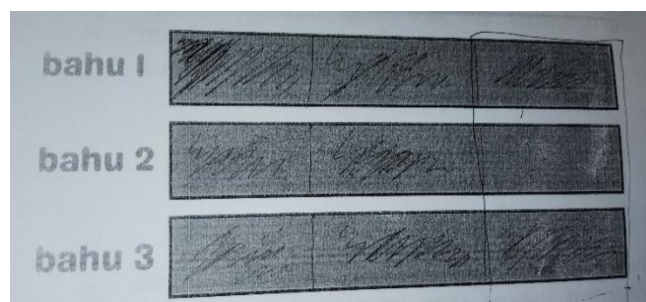

 Figure 3. Student's visual representation of shading $\frac{2}{3}$ of *bahu* units in Activity 2

Figure 3 displays one student's visual work when asked to shade $\frac{2}{3}$ of each bahu. The student divided each land strip into three parts and shaded two, showing an emerging strategy in visualizing fractional quantities. Such informal reasoning supports students in connecting intuitive ideas with formal mathematical understanding and real-world contexts.

Activity 3: Exploration with GeoGebra

This activity aims to guide students in exploring fractions dynamically using the GeoGebra applet. Students manipulate sliders for the numerator and denominator to see how the size of each part changes depending on the total number of divisions. They fill in a table showing the size of each part across different shapes (circle and square) and answer reflective questions about division, shape comparison, and fraction interpretation. This activity strengthens students' understanding of how fractions behave visually and symbolically, as well as their ability to generalize patterns.

Table 3. Comparison between HLT & ALT of activity 3

Learning Activity	HLT	ALT
Using a GeoGebra applet to explore fraction sliders (numerator and denominator).	<ol style="list-style-type: none"> 1. Students observe that changing the denominator makes parts smaller and changing the numerator shows how many are taken. 2. Students may misinterpret the visual, thinking the fraction is changing shape rather than part size. 	<ol style="list-style-type: none"> 1. Students correctly noted that the same fraction can be represented in different shapes. 2. Their table entries reflect a consistent understanding of part size.
Comparing different shapes	<ol style="list-style-type: none"> 1. Students realize that the same fraction looks different in circle and square, but the value is the same. 2. Students may assume different shapes give different results. 	<ol style="list-style-type: none"> 1. Students explained that the result remains the same because the numbers being divided are the same, even when the shape is different.
Explaining if dividing always leads to smaller parts	<ol style="list-style-type: none"> 1. Students might agree based on intuitive understanding that 	<ol style="list-style-type: none"> 1. Students agreed that dividing usually

	dividing makes things smaller.	results in smaller parts.
	2. Students might think that dividing can also mean re-grouping or re-representing, not just shrinking.	2. Student expressed that division does not always result in fair/equal parts
Understanding non-commutativity	1. Students recognize that dividing 3 by 2 gives a different result than dividing 2 by 3	1. Students correctly stated that $3 \div 2$ gives 1.5 and $2 \div 3$ gives 0.66

Table 3 presents a comparison between the Hypothetical Learning Trajectory (HLT) and the Actual Learning Trajectory (ALT) from Activity 3. These results provide insight into how students used GeoGebra to engage with dynamic fraction models and how their generalization of fraction structure began to develop. Although some students still required support in interpreting the symbolic implications of changes in the model, the integration of sliders and multiple representations effectively facilitated their exploration of mathematical patterns and relationships. One important interpretation is that the dynamic visualization demonstrated the RME principle of using models for progressive mathematization by bridging the students' contextual and visual understanding toward a more generic model.

Jumlah Bagian	Ukuran Tiap Bagian Lingkaran	Sisa	Ukuran Tiap Bagian Persegi	Sisa
2	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
3	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{3}$	$\frac{2}{3}$
4	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{4}$	$\frac{3}{4}$

Figure 4. A student completed the table in Activity 3 based on GeoGebra

Figure 4 presents one student's completed table after experimenting with the fraction model in GeoGebra. The student systematically recorded the size of each part and the remaining parts in both circular and rectangular representations. The student successfully identified a pattern in the relationship between the number of divisions and the corresponding fractional values. Even without a written explanation, the table entries show an emerging understanding of part-whole relationships. The use of GeoGebra allowed the student to observe changes

dynamically, supporting the development of conceptual generalization through exploration and trial.

Activity 4: Formalization of fraction notation

This activity aims to guide students in transitioning from visual and contextual representations to formal mathematical notation. Based on their prior exploration, students are asked to define the meaning of the numerator a and denominator b in a fraction $\frac{a}{b}$. Through guided reflection and articulation, students express that the numerator represents the number of parts taken or shaded, and the denominator represents the total number of equal parts. This activity develops students' symbolic understanding and solidifies the conceptual foundation of fraction structure.

Table 4. Comparison between HLT & ALT of activity 4

Learning Activity	HLT	ALT
Interpreting the roles of numerator and denominator in $\frac{a}{b}$	<ol style="list-style-type: none"> Students explain that a represents the number of parts taken or shaded, and b represents the total number of equal parts in the whole. Students confuse the roles by stating that a is the total quantity and b is the number used to divide. 	<ol style="list-style-type: none"> Students state that a is the shaded part, and b is the total number of parts (shaded and unshaded). Students state that a is the shaded part, and b is the unshaded part. Students state that a is all parts that have not been divided, and b is the divider. Students state that a is all the shaded parts, and b is the remaining unshaded parts.

Table 4 presents a comparison between the Hypothetical Learning Trajectory (HLT) and the Actual Learning Trajectory (ALT) from Activity 4. The findings demonstrate how students constructed symbolic representations of fractions by drawing on their prior contextual and visual experiences. Variations in students' responses underscore the inherent complexity of the formalization process. The iceberg model's facilitation of the transition from informal reasoning to formal notation exemplifies the RME principle of bridging informal and formal mathematical thinking.

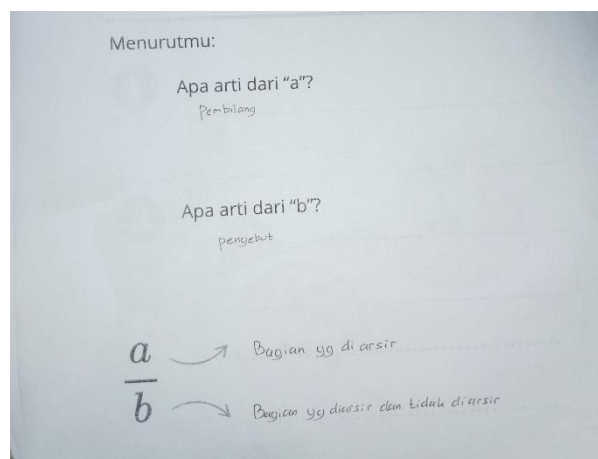


Figure 5. One student's response in Activity 4

Figure 5 shows one student's written response when asked to define the meaning of a and b in the fraction $\frac{a}{b}$. The student identified "a" as the shaded part and b as the total of the shaded and unshaded parts. The student's answer illustrates how contextual and visual learning activities contribute to meaning-making. At the same time, it confirms that some students still held partial misconceptions or required clarification, underscoring the importance of providing opportunities for further reflection and guided discussion in the formalization stage.

The results of the observation show that even though students are familiar with the concept of fractions, they experience confusion when applying it in the context of traditional land measurements, which they are not accustomed to encountering in their studies. When faced with contextual activities such as dividing 1 shoulder into $\frac{2}{3}$, students appear to need time to adjust their way of thinking. However, as the activity progresses, with guidance and visual support through GeoGebra, their understanding begins to develop. This demonstrates that realistic contexts, although initially challenging, can encourage students to develop a deeper understanding of concepts.

After completing all activities in the LKPD, students participated in structured interviews to share their reflections. Their responses offer insight into their learning experiences and perceptions of the materials used. Several students stated that the most interesting part of the LKPD was the use of GeoGebra and the task of dividing 1 bahu into $\frac{2}{3}$ by shading. Students reported that these activities were enjoyable and engaging, as they incorporated visual elements and concrete actions that facilitated comprehension of abstract mathematical concepts.

In the same activity, dividing 1 bahu into $\frac{2}{3}$ was also identified as the most difficult part of the LKPD by some students. Although students were engaged in the task, they experienced confusion when determining how to divide the unit accurately. To overcome this, some students used analogies, such as food or real-life objects, to visualize the process. For instance, one student described using the idea of “pizza” to explain the concept of fractions to their peers. This suggests that students naturally seek ways to ground mathematical ideas in concrete or familiar contexts when encountering difficulty.

Most students reported a better understanding of fractions after completing the LKPD. They felt more confident in explaining the meaning of fractions, particularly in terms of part-whole relationships using diagrams. However, not all students achieved the same level of clarity; one student explicitly stated that they still felt confused. This variation indicates that although the activities were generally effective, further support may be necessary for some students to fully internalize the concepts.

Regarding context, students agreed that the use of local units like *bahu* facilitated connecting mathematics with their everyday environment. They found it easier to conceptualize dividing familiar land, compared to abstract examples in textbooks. This supports the principle of realistic mathematics education, which emphasizes meaningful contexts to support conceptual understanding. Most students appreciated the visual and interactive approach of the LKPD; however, their preferences for instructional style varied. Some students expressed a preference for traditional instruction, including direct explanation of formulas by teachers. This highlights the importance of balancing visual, contextual, and procedural approaches to accommodate different learning preferences.

Discussion

The development of a Hypothetical Learning Trajectory (HLT) based on Realistic Mathematics Education (RME), integrated with GeoGebra, and the traditional land measurement context of Jombang, demonstrates that contextual and technological integration can significantly enhance students’ understanding of fractions. The findings reveal that when students engage with culturally familiar phenomena, they tend to reason more meaningfully and relate mathematical ideas to real-life situations. This finding aligns with research by Adha et al. (2024) and Fitriyana and Nursyahidah (2022), which indicates that incorporating local cultural contexts into mathematics education fosters student engagement and conceptual understanding.

An authentic cultural context serves as an effective phenomenological foundation for developing students' understanding of mathematical concepts. Student engagement with socially meaningful and familiar problems, such as fairness in land distribution, triggered the emergence of rich informal reasoning strategies. The students' ability to articulate the concept of "fairness" in quantitative terms indicates that real-life experiences can provide fertile ground for developing initial fraction concepts. This supports the RME principle of didactical phenomenology, where learning begins with familiar phenomena that are experientially real to students (Nursyahidah et al., 2025).

A key contribution of this study lies in demonstrating how the use of *bahu* serves as an effective phenomenological foundation for developing students' understanding of fractions. Student engagement with socially meaningful and familiar problems, such as fairness in land distribution, triggered the emergence of rich informal reasoning strategies. The students' ability to articulate the concept of fairness in quantitative terms indicates that real-life experiences can provide fertile ground for developing initial fraction concepts. This finding reinforces the RME principle of didactical phenomenology, which emphasizes that learning should start from phenomena that are experientially real to students (Nursyahidah et al., 2025). This finding is also consistent with studies by Adha et al. (2024) and Fitriyana and Nursyahidah (2022), which confirms that incorporating local contexts fosters student engagement and deeper conceptual understanding.

Furthermore, this study confirms the crucial role of representations as bridges to mathematical formalization. Observations revealed that student-constructed visual representations functioned as powerful mediating tools for connecting concrete experiences with abstract ideas. Visualizing land division and dynamically exploring the relationship between numerators and denominators in GeoGebra enabled students to form conjectures independently. This process embodies the RME principle of guided reinvention, where students are guided to reinvent mathematical ideas (Nursyahidah et al., 2024). This finding is supported by T. A. Putri et al. (2025), who emphasize that concrete and visual representations are essential bridges to mathematical abstraction and that technology can strengthen students' ability to construct meaning from observed patterns. The seamless transition from visual reasoning to symbolic notation $\frac{a}{b}$ observed at the end of the trajectory, was not a result of rote learning but a natural consequence of the meaningful understanding built from prior experiences, a process emphasized by Adha et al. (2024) as crucial for connecting informal reasoning with formal mathematical expressions.

Considering these findings, a major contribution of this study is its demonstration of how combining cultural context and digital technology creates a cohesive and effective learning trajectory. The *bahu* context provided a foundation of meaning and relevance, while GeoGebra offered a means for exploration, generalization, and pattern recognition. This synergy reinforces the RME principle that deep conceptual understanding is built by connecting the real-world contexts to mathematical abstraction through various representations (Nursyahidah et al., 2025), and aligns with reports by T. A. Putri et al. (2025) that technology integration enriches learning with dynamic representational support. This structured progression from informal to formal, as emphasized by R. I. I. Putri et al. (2025), is an essential element that must be grounded in real-life contexts.

Several limitations and challenges emerged during the implementation of this study. This study was conducted with a small group of students within a specific cultural context, which may limit the generalizability of the results. Practical challenges, such as limited computer access and the need for teacher facilitation, influenced the consistency of GeoGebra use. These limitations highlight that the effectiveness of RME-based HLTs depends not only on the design of contextual and digital integration but also on the readiness of teachers and the availability of technological infrastructure. Theoretically, this suggests that RME principles need to be continuously adapted to local learning environments, while in practice, future implementations should emphasize teacher capacity building and equitable access to technological resources. Future research could expand the implementation across diverse cultural and technological settings to refine the model and strengthen its theoretical and practical contributions.

Conclusion

This study successfully designed and implemented an RME-based HLT integrating GeoGebra with the traditional land measurement context of Jombang. The main finding demonstrates its significant effectiveness in enhancing students' conceptual understanding of fractions. Theoretically, this study contributes a validated prototype that concretizes the principles of Realistic Mathematics Education, providing a robust framework termed the Ethno-RME Design Framework for integrating digital tools with indigenous local wisdom to create culturally responsive learning models. Practically, the developed HLT serves as a useful guide for educators to design contextual and interactive mathematics lessons. For future research, two main directions are proposed. First, there is substantial potential to develop and formalize the Ethno-RME Design Framework into a more comprehensive theoretical model by exploring its core components and design principles in greater depth. Second, empirical studies should be conducted to test

and adapt this framework by replicating its application to other mathematical topics such as geometry, probability, or linear equations, and across diverse cultural settings. Additionally, longitudinal investigations are needed to explore the long-term impact of such integrated approaches on students' mathematical reasoning and knowledge retention.

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