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Development Hypothetical Learning Trajectory on Statistics Material in Grade VIII Using Realistic Mathematics Education at the Preliminary Stage with *Pranata Mangsa* Context

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Abstract

Statistics is a staple in the curriculum because it is considered necessary in everyday life. However, students often find statistics challenging. Students' difficulties in learning statistics are caused by several factors, including a lack of understanding of mathematical concepts and principles in solving problems. This study aimed to design and develop a Hypothetical Learning Trajectory (HLT) for grade VIII statistics using the Realistic Mathematics Education (RME) approach, with Pranata Mangsa as the learning context. The method used in this research was validation-type design research. This research produced an HLT of statistics material using *pranata mangsa* context consisting of 3 activities to help students understand and master the material. The activities of the HLT are 1) collecting, presenting, and analyzing data, 2) measuring centralized data, and 3) measuring data distribution. The results showed an HLT for teaching statistics using RME that can effectively support students of all ability levels. This HLT allows students to rediscover statistics concepts through horizontal and vertical mathematical processes. Finally, the HLT facilitates students to create their models from informal to formal and increases the interaction between students and teachers.

Keywords: Design Research; Hypothetical Learning Trajectory; *Pranata Mangsa;* RME; Statistics

Abstrak

Statistika menjadi materi pokok dalam kurikulum karena dianggap penting dalam kehidupan sehari-hari. Namun, pada kenyataannya statistika merupakan salah satu pelajaran yang dianggap sulit bagi siswa. Kesulitan siswa dalam belajar statistika disebabkan oleh beberapa faktor, diantaranya kurangnya pemahaman konsep dan prinsip-prinsip matematika dalam menyelesaikan soal. Tujuan penelitian untuk mendesain dan mengembangkan Hypothetical Learning Trajectory (HLT) menggunakan pendekatan Realistic Mathematics Education (RME) pada materi statistika kelas VIII menggunakan konteks Pranata Mangsa yang menjadi starting point dalam pembelajaran. Metode yang digunakan dalam penelitian ini adalah design research tipe validasi. Penelitian ini menghasilkan HLT materi statistika menggunakan konteks pranata mangsa yang terdiri dari 3 iceberg untuk membantu siswa dalam memahami dan menguasai materi. Adapun iceberg dari HLT tersebut adalah 1) mengumpulkan, menyajikan, dan menganalisi data; 2) ukuran pemusatan data; dan 3) ukuran penyebaran data. Hasil penelitian ini menunjukkan bahwa HLT mampu membantu siswa menemukan kembali konsep statistika melalui proses matematika horizontal dan vertikal. HLT memfasilitasi siswa untuk membuat model mereka sendiri dari informal ke formal dan meningkatkan interaksi antara siswa dan guru.

Kata Kunci: Design research; HLT; Pranata Mangsa; RME; Statistika

Introduction

Mathematics has been an essential subject since elementary school and helps students build their knowledge and understanding (Muah, 2022). Learning math is crucial because it trains students to think critically, rationally, and systematically when solving problems (Afriansyah et al., 2020; Pangestu & Hasti Yunianta, 2019). In addition, Hasibuan (2018) stated that mathematics is an expansive field and can be found in all aspects of life. Statistics is one of the branches of mathematics that is often found in everyday life.

Since statistics is thought to be essential in daily life, it is a mainstay of the curriculum (Febrianti & Chotimah, 2020). Statistics is the study of data collection, processing, analysis, and inference from the collected data set. Zhang (2018) states that the functions of using statistics include: 1) obtaining an overview of both a specific description and a general description of the state of an event; 2) following the development or ebb and flow of symptoms; 3) conducting tests; 4) gaining knowledge; 5) compiling reports in the form of quantitative data in an organized, concise, and transparent manner; 6) drawing conclusions logically, concluding precisely and steadily. However, in reality, statistics is one of the lessons that is considered difficult for students.

Maryati (2017) explained the difficulties experienced by students in learning statistics, namely: 1) difficulty in recognizing and classifying types of data and presenting them on graphs or tables; 2) difficulty in understanding what is asked; 3) difficulty in writing what is known in the problem and concluding. Similarly, Febrianti & Chotimah (2020), found that 83% of students struggle to connect mathematical ideas to diagrams, while 88% have difficulty interpreting story problems into diagrams or graphs. In addition, in statistical context problems related to statistics, students cannot recognize the mathematical relationship in the problem or solve it correctly (Monica & Retta, 2024).

Students' difficulties in learning statistics stem from several factors, including a lack of understanding of mathematical concepts and problem-solving principles (Ristiani & Maryati, 2022), low interest in mathematics (Hartono & Nursyahidah, 2021), and insufficient time allocated for solving problems (Maulana & Riajanto, 2021). Similarly, statistics is often taught as a collection of data and formulas, without opportunities for students to discover mathematical concepts independently. This approach hinders their ability to solve problems effectively.

To overcome this difficulty, a learning approach that is more contextual and involves students actively is needed. One of the relevant approaches is Realistic Mathematics Education. RME emphasizes the importance of using real contexts for students to rebuild mathematical concepts through the exploration of everyday problems (Andzin et al., 2024; Hardiyanto et al., 2024; Nursyahidah, 2021; Nursyahidah et al., 2020). This approach connects students and improves their understanding of statistical concepts (Munir & Sholehah, 2020).

However, for RME to be implemented effectively, a structured learning plan is required (Wijaya et al., 2021). One of the supporting strategies is the development of a Hypothetical Learning Trajectory. HLT is a guide that designs learning activities to achieve specific goals while helping teachers evaluate and adjust teaching (Ulfa & Wijaya, 2019). Research by Silvia et al. (2021) showed that using HLT in RME-based learning can improve students' mathematical problem-solving compared to conventional learning.

This research used *pranata mangsa* as the initial context in learning statistics for grade VIII students. *Pranata mangsa* is a traditional Javanese calendar system used by farmers and fishermen to determine the time for planting and fishing. *Pranata mangsa* aimed to introduce statistical concepts through existing data and local cultural heritage, potentially increasing students' sense of relevance and connection to learning.

Although not all students were familiar with *pranata mangsa*, this context was chosen because it is authentic and can be adapted into data-driven learning models, such as observing seasonal patterns or trends at a particular time. With the proper guidance, students could understand basic statistical concepts such as data collection, processing, and analysis through a meaningful context. In addition, the use of *pranata mangsa* supported the ethnomathematics approach, which was aligned with the principles of RME. This research was limited to the preliminary design stage to ensure that the basic concepts and assumptions of the HLT were valid and applicable before proceeding to further stages.

Based on this description, this research aimed to design and develop HLT using the RME approach on statistics material in grade VIII, utilizing *pranata mangsa*'s context as a learning starting point.

Method

This study used the validation-type design research method, which was a type of design research that aimed to develop learning trajectories through collaboration between researchers and teachers to improve the quality of learning (Akker et al., 2006). This method focused on validating and improving learning designs through iterative research cycles.

The stages in design research, according to (Gravemeijer & Cobb, 2006), include:

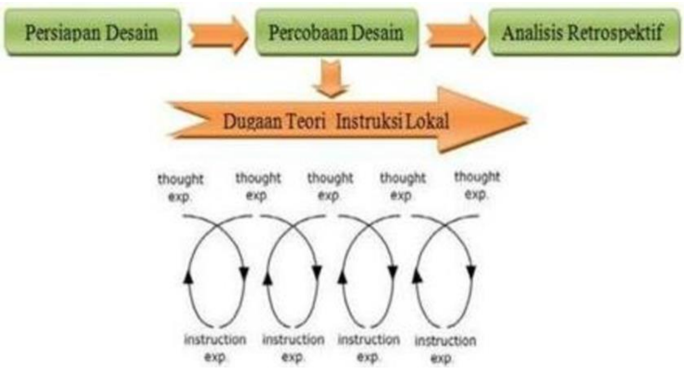


Figure 1. Design Research Validation-Type

1. Preparing for the Experiment: Designing the learning sequence and methods to assess the learning process.
2. Design Experiment: The pilot and teaching experiments are conducted in two stages.

3. Retrospective Analysis: Review the experiment results to refine the learning design.

This research was limited to the preliminary design stage, and conducted in preparation for the experimental stage. The goal was to design the learning sequence and assess the learning process. In this stage, a literature review related to Hypothetical Learning Trajectories, RME, statistics material, and Ethnomathematics was conducted. The HLT was designed as an iceberg model, a flexible model that serves as a guide that can be adjusted experimentally for each learning activity. The subjects in this study were 38 VIII-grade students with different characteristics from one of the junior high schools in Semarang.

The data sources utilised in this investigation were derived from pertinent papers or journals. The data collection techniques were written tests, observation, interviews, and documentation. In comparison, the data analysis technique used was the Miles and Huberman model analysis. In this model, activities were carried out interactively and continuously until sufficient. Data review included reduction, display, and conclusion drawing/verification.

Data reduction involved summarizing, identifying, and prioritizing essential elements related to the research topics. The data obtained from these documents was extensive, so it needed to be summarized in greater detail. The data obtained from these documents was extensive, so it needs to be summarized in more detail. Therefore, it was necessary to analyze the data immediately by reducing it.

The next step was to present the data, which came from analytical conclusions. Presenting the data helped the researcher better understand issues related to HLT development, including its use with the RME approach and its application to statistics material using the *pranata mangsa* context.

The final step of data analysis was to take opinions and evidence. The new findings were the conclusion of the existing findings. The initial conclusions were still provisional and would change if no solid supporting evidence was found at the following data collection stage.

Results

The development of the HLT during the preparation for the experiment stage began with an analysis of curriculum literature. This step ensured that the designed learning aligned with the grade VIII mathematics curriculum, including teaching materials, learning objectives, and learning indicators.

Gravemeijer (1994) suggests three main principles in RME: guided reinvention/progressive mathematizing, didactical phenomenology, and self-developed models. Guided reinvention refers to the process where students, with guidance from the teacher, "reinvent" mathematical concepts. Progressive mathematizing involves a gradual process of mathematization, where students move from real-world situations to abstract mathematical thinking. In the RME approach, students solve contextual problems at the start of learning, and the teacher provides limited guidance, allowing students to work through issues constructively. This process helps students rediscover mathematical concepts, principles, properties, and formulas, simulating independent learning.

Didactical Phenomenology: This principle focuses on selecting mathematical topics based on two critical criteria: 1) to highlight the broad range of applications to be learned and 2) to assess whether the topic can contribute effectively to the progressive mathematization process. In other words, the subject matter should be chosen for its relevance to the real world and for how it supports the gradual development of mathematical thinking.

Self-Developed Models: This principle bridges the gap between informal and formal knowledge. It encourages students to create problem-solving methods, allowing them to construct models and solutions based on their understanding. This approach empowers students to take ownership of their learning, fostering a deeper, more personalized grasp of mathematical concepts.

The concept of an iceberg can be used to show the different ways that mathematical representations relate and are used to aid mathematical learning regarding formal mathematical goals. The tip of the iceberg represents the goal. This is usually the formal representation. At the bottom of the tip of the iceberg is a much larger section containing many informal and pre-formal mathematical representations (models of and models for) that relate to the formal mathematical goal. These iceberg mathematical representations are also organized in a particular way. First, the context supports students' informal reasoning at the bottom of the iceberg. Then, pre-formal models, representations, and strategies related to the goal are placed above the informal level in the center of the iceberg. Basically, below the tip of the iceberg is the substance, i.e., the "floating capacity" of the iceberg, to help understand the mathematics depicted at the top. This floating capacity includes reasoning with related contexts (informal) and using models, tools, representations, and strategies (pre-formal). The results obtained in this study were in the form of HLT design, in which there was a learning trajectory for statistics material using *pranata mangsa* as a starting point for learning or context.

The initial design stage resulted in developing a HLT represented as a learning trajectory design. This study used the context of seasonal patterns in *pranata mangsa* to improve students' understanding of statistical concepts.

As part of the learning, students were introduced to a context video containing a brief explanation of *pranata mangsa*, including its history, function, and relevance in the life of an agrarian society. The video also presented data on *pranata mangsa*, such as seasonal patterns, time division in a year, and types of agricultural activities adjusted to the season. Students used this data to collect, present, and analyze information as part of the problem-solving process in everyday life.

Students who were not farmers or fishermen can still engage with the learning material, as the necessary data was provided in the video. Thus, all students, regardless of their background, understood and processed the data in the statistical context being taught.

Through this learning trajectory, students were directed to calculate data centering and dispersion measures, such as mean, median, mode, range, quartile, quartile range, and quartile deviation. In addition, students also solved problems related to data concentration and distribution measures based on the data presented in the video. The HLT design for statistics material with the context of Pranata Mangsa can be seen in Figure 2.

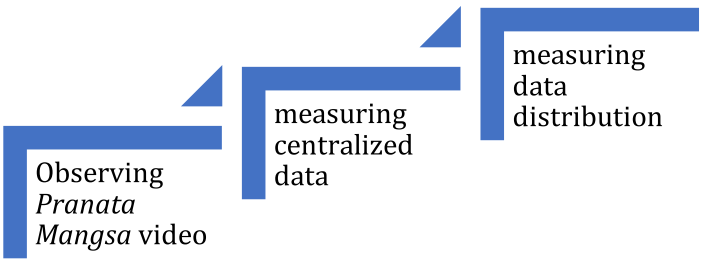
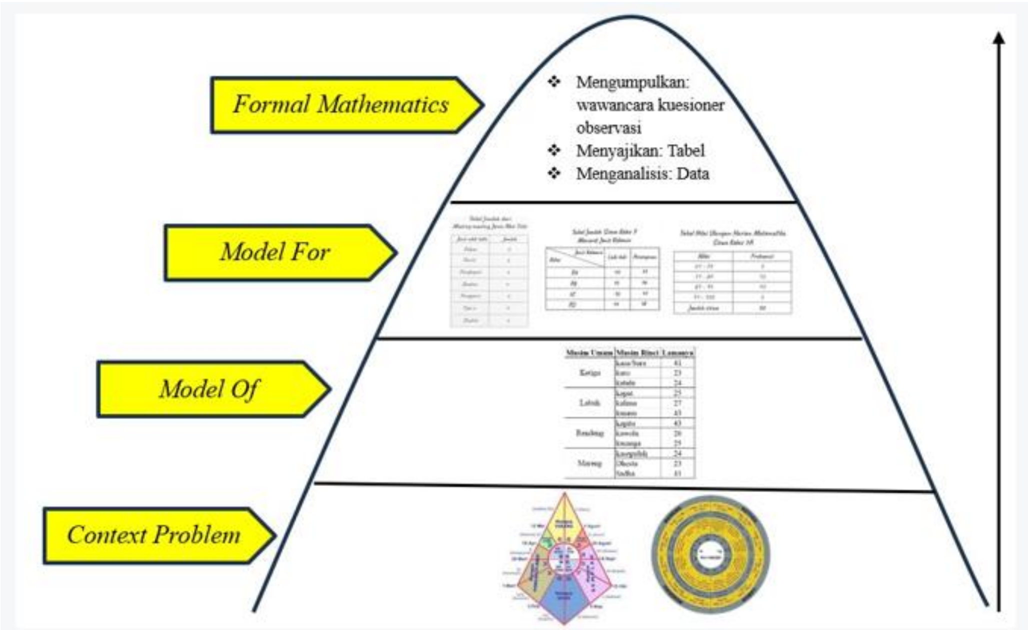


Figure 2. HLT of statistics material

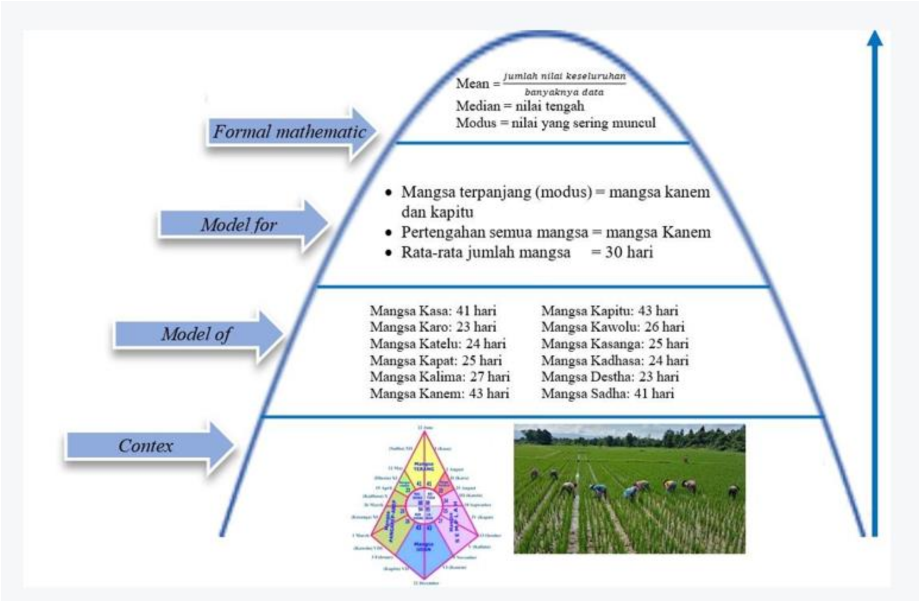
The hypothetical learning trajectory developed in this study consists of three activities, namely: (1) observing the *pranata mangsa* context, (2) measuring centralized data, and (3) measuring data distribution. Each activity designed from informal to formal level can be described through the iceberg. For the first activity, observing *pranata mangsa* context can be seen in Figure 3.



Formal Mathematics	Presenting data
Model For	Sorting and categorizing data
Model Of	Collecting data from *pranata mangsa* videos
Context Problem	Observing the data contained in the *pranata mangsa* video

Figure 3. An Iceberg of Collecting, Presenting, and Analyzing Data

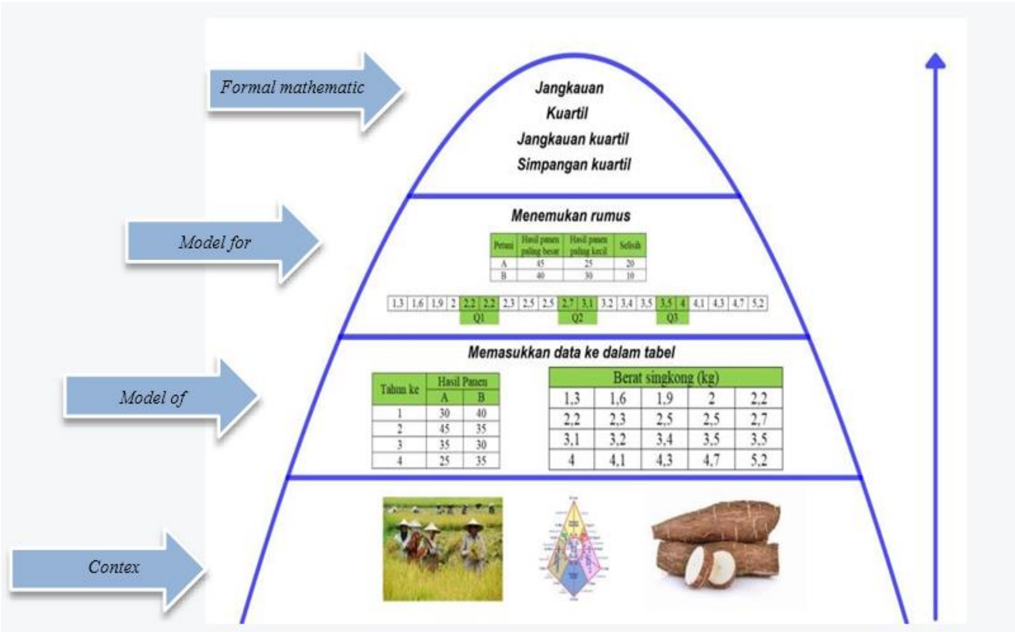
Based on Figure 3, the goal of the first activity was to collect and present data from the context provided in the video. This activity conjectured that students could gather and present data and solve a contextual problem based on these tasks. Then, the design of activity two can be described in Figure 4.



Formal Mathematics	Data centering measures
Model For	Find frequent data, centered data, and average data.
Model Of	Collecting data from pranata mangsa videos
Context Problem	Observing the data contained in the *pranata mangsa* video

Figure 4. An iceberg of Data-Centralized Measures

Figure 4 illustrates activity 2, which was designed in this study to transition students from solving contextual problems to formal mathematical concepts. This activity aimed to determine the mean, median, and mode formula using the *pranata mangsa* context. From this second activity, it was hoped that students could find means, medians, and modes and solve contextual problems related to those materials. Furthermore, the design of Activity 3 can be described in Figure 5.



Formal Mathematics	Measures of data spread
Model For	Find the difference between the most significant and minor values and divide the data into four parts.
Model Of	Collecting data from *pranata mangsa* videos
Context Problem	Observing the data contained in the *pranata mangsa* video

Figure 5. An Iceberg Measure of Data Distribution

Figure 5 illustrates activity 3, designed in this study from solving contextual problems to understanding formal mathematical concepts. This activity aimed to determine the formula of range, quartile, quartile range, and quartile divergence using the *Pranata Mangsa* context. From this activity, it was hoped that students could solve contextual problems related to those materials.

At the bottom of Figure 3, Figure 4, and Figure 5 are examples of contexts that can elicit students' informal reasoning. Students were often taught mathematical procedures without opportunities to connect them to relatable real-world contexts, such as the Pranata Mangsa calendar. Perhaps students would not be able to understand mathematical thinking if they were only given a picture. However, if questions were added "What *mangsa* was the longest in duration? What season was the middle of the *mangsa*? What was the average day of the *mangsa*?" there will be a math problem that students could solve.

Students considered the problem of determining which season lasted the longest. Low-ability students likely calculated the days of each season one by one. Students with moderate abilities likely performed more complex calculations than those with lower abilities. Students with higher abilities likely compiled data on the number of days for the six types of *mangsa* presented. This happens in the pre-formal stage of the pre-formal stage.

Next to the higher level, namely the “model for” stage. Low, medium, and high-ability students had determined the most extended season in *pranata mangsa*, the season in the middle of the time, and the average day in *pranata mangsa*. Students used this information to move to the next stage, namely the pre-formal stage (model of and model for), in various ways that could be taken according to students' abilities and creativity. The stage is one of the students' thoughts. At the top of the iceberg is formal knowledge reflecting the purpose of learning data concentration. Meanwhile, the part that develops underneath includes steps to help students reach the formal mathematics stage.

Discussion

Students' understanding of statistical concept could be supported by learning trajectories with the RME approach using the context of *pranata mangsa*. Students' knowledge could be developed from informal to formal stages. The design of learning activities could change students' views that learning mathematics was considered difficult because it was far from the context of everyday life. However, it became evident that mathematics was rooted in everyday contexts and could be applied meaningfully. RME approach with Javanese culture *Pranata Mangsa* was used as a context in learning statistics material. Some previous studies used local contexts in learning mathematics, such as puppet stories and *uno stucko* in learning number patterns (Risdiyanti & Prahmana, 2020); Indigenous traditions for cone learning (Nursyahidah et al., 2020); and Indonesian handicrafts (wicker) in learning measurement and geometry (Aklimiwati et al., 2022).

Integrating the context of *pranata mangsa* into learning statistics not only increased student interest and motivation but also helped solve real-world problems. Students were encouraged to apply statistical concepts to analyze agricultural cycles. They applied the traditional calendar system as a context, weather patterns, or other relevant topics. As Widiawati et al. (2018) highlighted, the RME approach facilitates student understanding of concepts and boosts engagement. This approach also fostered collaboration, where students worked in discussion groups to solve problems. As a result, the learning process became more meaningful, and the issues were resolved effectively through both conceptual mastery and practical application.

Conclusion

This research produced an HLT of statistics material using the context of *pranata mangsa,* which consisted of 3 activities to help students understand and master the material. The series of activities in the HLT were 1) collecting, presenting, and analyzing data, 2) determining the size of data concentration, and 3) determining the size of data distribution. The created iceberg design could be further elaborated into a learning trajectory to enhance student comprehension of statistical topics and was relevant during the teaching experiment phase. The results showed an HLT for teaching statistics using RME that could help all levels of students, and it can be improved to help students based on their cognitive style or learning style. This HLT allowed students to rediscover statistics concepts through horizontal and vertical mathematical processes. Finally, the HLT facilitated students in creating their models from informal to formal and increased the interaction between students and teachers.

References

Afriansyah, E. A., Herman, T., Turmudi, T., & Dahlan, J. A. (2020). Mendesain soal berbasis masalah untuk kemampuan berpikir kritis matematis calon guru. *Mosharafa: Jurnal Pendidikan Matematika*, *9*(2), 239–250. https://doi.org/10.31980/mosharafa.v9i2.649

Akker, J. van Den, Gravemeijer, K., McKenney, S., & Nieveen, N. (2006). *Educational Design Research*. Routledge.

Aklimiwati, Listiana, Y., Isfayani, E., & Zainuddin, R. A. (2022). Pengembangan hypothetical learning trajectory (HLT) berbasis realistic mathematics education (RME) pada materi geometri. *Jurnal Serunai Matematika*, *14*(2), 51–64.

Andzin, N. S., Sari, P. Y. P., Widodo, R. C., Sukowati, D. I., Indriastuti, S., & Nursyahidah, F. (2024). Arithmetic sequences and series learning using realistic mathematics education assisted by augmented reality. *Jurnal Pendidikan Matematika*, *18*(1), 139–148. https://doi.org/10.22342/jpm.v18i1.pp139-148

Febrianti, V., & Chotimah, S. (2020). Analisis kesulitan pada materi statistika kelas VIII siswa SMP. *Jurnal Pembelajaran Matematika Inovatif*, *3*(5), 559–566. https://doi.org/10.22460/jpmi.v3i5.559-566

Gravemeijer, K. (1994). *Mengembangkan Pendidikan Matematika Realistik*. Institut Freudenthal.

Gravemeijer, K., & Cobb, P. (2006). *Design research from a learning design perspective, in Educational Design Research*. Oxfordshire.

Hardiyanto, D., Asokawati, I., Majid, P. M., Maesaroh, A. T., & Nursyahidah, F. (2024). Learning reflection using realistic mathematics education assisted by GeoGebra software. *Jurnal Pendidikan Matematika*, *18*(1).

Hartono, H., & Nursyahidah, F. (2021). Supporting 7th grade students' understanding of angles using Central Java traditional house. *Kreano, Jurnal Matematika Kreatif-Inovatif*, *12*(2), 212–226. https://doi.org/10.15294/kreano.v12i2.30541

Hasibuan, E. K. (2018). Analisis kesulitan belajar matematika siswa pada pokok bahasan bangun ruang sisi datar di SMP Negeri 12 Bandung. *AXIOM: Jurnal Pendidikan Dan Matematika*, *7*(1), 18–30. https://doi.org/10.30821/axiom.v7i1.1766

Maryati, I. (2017). Analisis kesulitan dalam materi statistika ditinjau dari kemampuan penalaran dan komunikasi statistis. *Prisma*, *6*(2), 173–179. https://doi.org/10.35194/jp.v6i2.209

Maulana, P. A., & Riajanto, M. L. E. J. (2021). Analisis kesulitan pembelajaran daring siswa MTS kelas VIII pada materi statistika di masa pandemi covid-19. *JPMI : Jurnal Pembelajaran Matematika Inovatif*, *4*(5), 1393–1404. https://doi.org/10.22460/jpmi.v4i5.1393-1404

Monica, R., & Retta, A. M. (2024). *Kesulitan dalam menyelesaikan soal AKM pada materi statistika di kelas VIII SMP*. *6*(1), 116–126.

Muah, T. (2022). Penerapan pendekatan pembelajaran Realistic Mathematics Education (RME) untuk meningkatkan hasil belajar matematika materi segiempat. *Paedagogia*, *25*(2), 132. https://doi.org/10.20961/paedagogia.v25i2.64545

Munir, M., & Sholehah, H. (2020). Pembelajaran matematika realistik Indonesia (PMRI) dalam meningkatkan kemampuan pemecahan masalah. *Jurnal Al-Muta'aliyah*, *5*(1), 33–42.

Nursyahidah, F. (2021). Learning design on surface area and volume of cylinder using Indonesian Ethno-mathematics of traditional cookie maker assisted by GeoGebra. In *Mathematics Teaching-Research Journal* (Vol. 13, Issue 4, pp. 79–98).

Nursyahidah, F., Saputro, B. A., Albab, I. U., & Aisyah, F. (2020). Pengembangan learning trajectory based instruction materi kerucut menggunakan konteks megono gunungan. *Mosharafa: Jurnal Pendidikan Matematika*, *9*(1), 47–58. https://doi.org/10.31980/mosharafa.v9i1.560

Pangestu, N. S., & Hasti Yunianta, T. N. (2019). Proses berpikir kreatif matematis siswa extrovert dan introvert SMP kelas VIII berdasarkan tahapan Wallas. *Mosharafa: Jurnal Pendidikan Matematika*, *8*(2), 215–226. https://doi.org/10.31980/mosharafa.v8i2.472

Risdiyanti, I., & Prahmana, R. C. I. (2020). The learning trajectory of number pattern learning using barathayudha war stories and uno stacko. *Journal on Mathematics Education*, *11*(1), 157–166. https://doi.org/10.22342/jme.11.1.10225.157-166

Ristiani, A., & Maryati, I. (2022). Kemampuan representasi matematis dan self-esteem siswa pada materi statistika. *Jurnal Inovasi Pembelajaran Matematika: PowerMathEdu*, *1*(1), 37–46. https://doi.org/10.31980/powermathedu.v1i1.1914

Silvia, S., Fauzan, A., Musdi, E., & Jamaan, E. Z. (2021). Pengembangan desain pembelajaran statistika berbasis Realistic Mathematic Education (RME). *AKSIOMA: Jurnal Program Studi Pendidikan Matematika*, *10*(4), 2849–2858. https://doi.org/10.24127/ajpm.v10i4.4362

Ulfa, C., & Wijaya, A. (2019). Expanding hypothetical learning trajectory in mathematics instructional. *Journal of Physics: Conference Series*, *1320*, 1–7. https://doi.org/10.1088/1742-6596/1320/1/012091

Widiawati, Marzal, D., & Juwita, H. (2018). Desain pembelajaran garis dan sudut dengan konteks pagar buluh di kelas VI. *Journal of Mathematics Science and Education*, *1*(1), 118–130. https://doi.org/10.31540/jmse.v1i1.186

Wijaya, A., Elmaini, E., & Doorman, M. (2021). A learning trajectory for probability: A case of game-based learning. *Journal on Mathematics Education*, *12*(1), 1–16. https://doi.org/10.22342/jme.12.1.12836.1-16

Zhang, Y. (2018). Application of project teaching method on statistical analysis teaching. *Proceedings of the 6th International Conference on Social Science, Education and Humanities Research (SSEHR 2017)*, *185*, 253–256. https://doi.org/10.2991/ssehr-17.2018.59.

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