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Content of School Mathematics Textbook: A Praxeological Analysis on Indefinite Integral

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Abstract

Indefinite integrals are a fundamental concept in calculus. This qualitative study aims to describe the content of school mathematics textbooks on indefinite integrals. The analysis used the concept of praxeology, which includes theory, technology, and techniques, based on the Reference Epistemological Model. The primary instrument in this research was the researcher, who used data from school mathematics textbooks, including two electronic textbooks used in Indonesia, one textbook from Singapore, and one from the Cambridge curriculum. The study results included a Reference Epistemological Model for indefinite integrals, used to compare textbook content and praxeological components. Praxeological analysis shows that the theoretical and technological components of Indonesian school mathematics textbooks are more complex than those from Singapore and the Cambridge curriculum. These differences are attributed to variations in the mathematics curricula of the three regions. Therefore, the Reference Epistemological Model can guide the development of instructional materials on indefinite integrals, taking into account the complexity differences among the textbooks.

Keywords: Indefinite Integral; Praxeological; Reference Epistemological Model.

Abstrak

Materi integral adalah salah satu konsep fundamental dalam kalkulus. Penelitian ini merupakan penelitian kualitatif yang bertujuan untuk mendeskripsikan muatan pada buku teks matematika sekolah pada materi integral tak tentu. Analisis menggunakan konsep prakseologi yang meliputi teori, teknologi, dan teknik berdasarkan Referensi Model Epistemologi. Instrumen utama dalam penelitian ini adalah peneliti sendiri dengan menggunakan sumber data berupa buku teks matematika sekolah yaitu dua buku sekolah elektronik yang digunakan di Indonesia, satu buku teks matematika yang digunakan di Singapura, dan satu buku teks kurikulum Cambridge. Hasil penelitian berupa Referensi Model Epistemologi pada materi integral tak tentu yang digunakan untuk perbandingan buku dan organisasi prakseologi. Analisis prakseologi menunjukkan bahwa komponen teori, teknologi, dan teknik pada buku teks matematika sekolah dari Indonesia lebih kompleks dibandingkan dengan buku dari Singapura dan kurikulum Cambridge. Adanya perbedaan tersebut didasarkan pada muatan kurikulum yang berbeda diantara ketiganya. Dengan demikian, Referensi Model Epistemologi yang dihasilkan dapat dijadikan salah satu acuan dalam mengembangkan pembelajaran pada materi integral tak tentu, termasuk perbedaan kompleksitas antar kedua buku dapat menjadi referensi untuk digunakan sebagai sumber pembelajaran.

Kata Kunci: Integral Tak Tentu; Prakseologi; Referensi Model Epistemologi.

Introduction

Calculus is one of the branches of mathematics that has an important role in life. At the level of science, calculus is the foundation and gateway for the development of more advanced mathematics (Tall, 1997). In addition, calculus concepts are widely used in helping solve science and technology problems, in addition to solving using algebra (Carlson et al., 2010; Oktaviyanti & Supriani, 2015; Usman et al., 2019). In the education curriculum in Indonesia, Calculus is given in mathematics lessons in Senior High School (Cheshier, 2006), thus providing a foundation of understanding to students as a form of preparation for studying calculus at the University level (Blank, 1960; Robinson, 1970).

In outline, calculus material studied at the high school level includes concepts related to limit functions, derivatives, and indefinite and definite integrals. Integral is one of the calculus materials studied by high school students. In this material, students learn the integral as an anti-derivative. In the integral material, students are expected to have the ability to: (1) calculate indefinite integrals on algebraic and trigonometric functions; (2) calculate definite integrals on algebraic and trigonometric functions; and (3) apply integral concepts in calculating the area of flat areas and the volume of rotating solid objects.

The reality in the field shows that students' ability to solve calculus problems in general, as well as specifically on integral concepts is still low. This is based on several research results (Amelia & Yadrika, 2019; Fadillah et al., 2019; Nurhikmah & Febrian, 2016; Rahmi et al., 2020; Usman et al., 2019). The results of these studies show that students still experience difficulties and errors in understanding and solving integral problems, which include concept errors, fact errors, principle errors, and operation errors. It is further stated that these errors are based on students' lack of understanding of the integral concept as a whole. In addition to cognitive aspects, students' low ability to understand integral concepts is also based on students' low interest in learning integral material (Fadillah et al., 2019).

Given the importance of the calculus concept as stated above, this condition must certainly be resolved immediately. Brewer and Stacz stated that several factors affect student performance in learning, namely the curriculum, textbooks, and teaching practices carried out by teachers (Wijaya, 2019). At the level of the learning process, several studies have shown very good things through the use of technological devices or learning media in learning calculus material, especially integrals (Mahayukti & Dewi, 2021; Pramuditya et al., 2019; Yuniarti et al., 2018; Zakaria & Salleh, 2015). Orton, (1985) highlighted that calculus learning, including integral concepts, requires the most careful introduction and development, so learning calculus without proper planning will be risky for students' future understanding.

Seeing these conditions, of course, it will be very necessary to see how the learning process occurs. Although there are several research results as alternative solutions for learning integral calculus using technological devices, these alternatives are completely unworkable, especially in schools that do not have adequate technological device facilities. Thus, the learning process is still fully centered on using teaching materials such as textbooks as learning resources without using technological devices.

Textbooks as one of the main references in learning must be textbooks that can provide students with a correct understanding of concepts. Ain and Kurniawati stated that it is undeniable that the dependence of teachers (Rizqi et al., 2021)and students on textbooks is very large, both as a source of material and practice questions (Tanujaya et al., 2017). Based on this, the selection of the right mathematics textbook is very important, so that students get a comprehensive understanding.

In the Indonesian context, the development of school textbooks including those for mathematics has been facilitated through the Pusat Kurikulum dan Perbukuan of the Ministry of Education, Culture, Research and Technology. In its development, school mathematics textbooks have also undergone adjustments to the learning objectives and the applicable national curriculum, to provide equal opportunities for each school in improving student abilities. On the other hand, it is undeniable that there are also schools in Indonesia that adopt curricula from other countries, such as IB, Cambridge, and so on, including from Singapore. The differences in curriculum have an impact on learning objectives that are aligned with the use of textbooks. In mathematics, several studies (Lisarani et al., 2018; Ramelan & Wijaya, 2019; Yang & Sianturi, 2017) state that the objectives of the mathematics curriculum in Indonesia emphasize the use of science in learning, while in Singapore on problem-solving. Explained by Hoong et al. (2015) that mathematics textbooks in Singapore are developed based on the Concrete-Pictorial-Abstract (CPA) framework which refers to Bruner's enactive-iconic-symbolic learning theory. This can certainly provide a new reference for the development of mathematics textbooks in Indonesia.

Several studies related to the analysis of mathematics textbooks have been conducted (Muniarti et al., 2021; Ramda et al., 2018). Research related to textbook analysis, especially regarding integral calculus material has not yet been carried out. In addition, the focus of existing research is only based on curriculum analysis, while the focus on material or concepts is still rarely done. On the other hand, the analysis of textbooks, including mathematics textbooks, can have an impact on the achievement of students' mathematical abilities (Aminah & Kurniawati, 2018; Malihatuddarojah & Prahmana, 2019). By referring to some of the previously presented research results related to student mastery of indeterminate integral material (Amelia & Yadrika, 2019; Fadillah et al., 2019; Nurhikmah & Febrian, 2016; Rahmi et al., 2020; Usman et al., 2019) then analyzing mathematics textbooks on indeterminate integral material needs to be done as an alternative to overcome these problems.

Textbook analysis that focuses on concepts, one of which, is based on one of the branches of epistemology (Rizqi et al., 2021), namely the praxeology organization model which contains four elements, namely the *Type of Task, Technique, Technology*, and *Theory*. Praxeology is part of the Anthropological Theory of the Didactic (ATD) presented by Chevallard (Wijayanti & Winslow, 2017). The Anthropological Theory of the Didactic is a theory that observes human mathematical activity through an epistemological model of mathematical knowledge (Chevallard, 1992). Praxeology consists of *Praxis* block or practical block and *Logos* block or knowledge block (Wijayanti & Winslow, 2017) as in the following Figure 1.

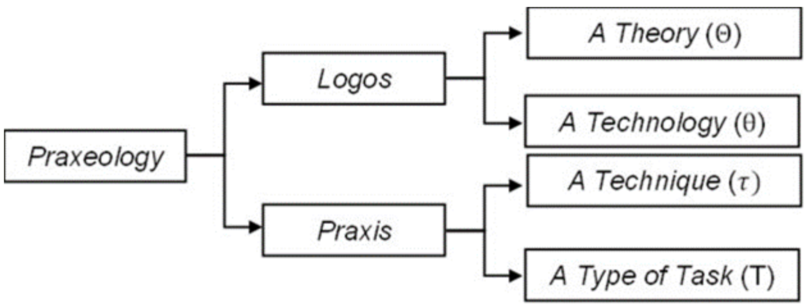


Figure 1. Praxeology Model

A *Type of task* (T) is a specific problem given to students. In the classroom learning process, these problems can be given through textbooks. To solve the given problem, students need a *Technique* (τ). Between T and τ is a one-to-one correspondence, that is, T consists of several tasks that can be solved by τ (Wijayanti & Winslow, 2017). Furthermore, the logos block consists of *Technology* (θ) and *Theory* (Θ). *Technology* (θ) is a justification or argumentation of the *techniques* used by students in completing a task. Furthermore, *Theory* (Θ) is a generally applicable concept in mathematics to justify the various technologies used.

It is further explained by Putra and Witri (2017) that most *types of tasks* can be completed by various *techniques*. A *technology* can consist of several *techniques* and a set of *types of tasks* and *techniques* for completing these tasks is referred to as a punctual organization. Furthermore, a general *technology* to justify several *techniques* is called a local organization. A *theory* that is used for several *technologies* is called a regional organization. The following in Table 1 is an example of a pre-technology organization according to Wijayanti and Winslow (2017).

Table 1. Example of Praxeological Organization

Praxeology Components	Example
Theory	Proportions
Technology	Direct proportion: Ration and scale
Technique	$r = \frac{x_1}{x_2}$
Type a Tasks	Given x_1 and x_2 so that $(x_1, x_2) \sim (1, r)$

Furthermore, in conducting analysis activities, a Reference Epistemological Model (REM) is used which shows some activities that simply describe explicitly the description of the elements of praxeology, namely *type of task, technique, technology*, and *theory*. Given the importance of calculus material, including indeterminate integral material, in students' mastery of mathematics, students should be able to master the material. The presentation of concepts in mathematics textbooks is crucial for enhancing student understanding. Therefore, it should be a primary concern in the learning process. This shows the need for research that examines how the content of school mathematics textbooks for indefinite integral material is presented. Thus, this study was conducted to analyze the content of school mathematics textbooks on school materials using the organization of praxeology, namely *Theory* (Θ), *Technology* (θ), and *Technique* (τ).

Method

This research was a descriptive qualitative that aims to describe the content in mathematics textbooks, especially on integral material based on praxeological theory. The data in this study were qualitative data obtained from reference books and mathematics textbooks as data sources. The main instrument in this research was the researcher by considering the Reference Epistemological Model (REM) as an auxiliary instrument. The auxiliary instrument in this research was a checklist containing suitability criteria based on the epistemological referential model which contains elements of *technique*, *technology*, and *theory*. The data collection technique used in this research was the documentation method which is carried out by analyzing book content. The descriptive data was analyzed using the Praxeology Model.

The research began by selecting mathematics textbooks that cover indefinite integral materials. The mathematics textbooks analysed in this study were mathematics electronic schoolbooks (BSE) published by the Ministry of Education, Culture, Research, Technology and Higher Education, mathematics textbooks used in Singapore, and textbooks based on the Cambridge curriculum. BSE books published by the Ministry were chosen because they are the main reference and source of learning in all schools in Indonesia. Furthermore, mathematics textbooks were used in Singapore between 2013 and 2017. The math textbook was chosen because it has been approved for use in learning by the Singapore Ministry of Education. The selection of textbooks from Singapore is based on the objectives of the mathematics learning curriculum which emphasizes problem-solving so that it can be one of the references in Indonesia (Lisarani et al., 2018; Ramelan & Wijaya, 2019; Yang & Sianturi, 2017). The following is the identity of the book in question as in the following Table 2.

Table 2. Identity of Mathematics Textbooks

Authors	Title	Publisher	Year	Code
Sudianto Manullang, Andri Kristianto S., Tri Andri Hutapea, Lasker Pangarapan Sinaga, Bornok Sinaga, Mangaratua Marianus S., & Pardomuan N.J.M Sinambela	Matematika untuk SMA/MA/SMK/MAK Kelas XI	Pusat Kurikulum dan Perbukuan, Balitbang, Kemendikbub	2018	B1
Pesta E.S., & Cecep Anwar H.F.S.	Matematika Aplikasi: Untuk	Pusat Perbukuan,	2008	B2

Authors	Title	Publisher	Year	Code
	SMA dan MA Kelas XII Program Studi Ilmu Alam	Departemen Pendidikan Nasional		
Joseph Yeo, Teh Keng Seng, Loh Cheng Yee, & Ivy Chow	New Syllabus Additional Mathematics 9th Edition	Shinglee Publishers PTE LTD	2013	B3
Tony Beadsworth	Complete Additional Mathematics for Cambridge IGCSE & O Level	Oxford University Press	2017	B4

In addition, a content analysis of mathematics textbooks related to integral material would be used as a reference in compiling REM, namely Calculus books used at the University level with the title Calculus Ninth Edition with the authors Varberg, Purcell, and Rigdon in 2007 and Calculus Eighth Edition with the author Stewart in 2016. The epistemology model reference was used as a reference in analyzing mathematics textbooks used in schools based on the praxeology model.

The data analysis technique was carried out in the following stages: (1) data reduction, which involved summarizing and selecting the main elements to be analyzed from the textbooks used by schools, then comparing them with the Reference Epistemological Model (REM) prepared previously; (2) data presentation, which involved presenting the data in narrative form, tables, or charts to show the overall connection between the condition of the school mathematics textbooks based on the comparison with the Reference Epistemological Model; and (3) verification and conclusion drawing, which involved verifying the data obtained and then drawing conclusions based on the results of the discussion and verification (Miles & Huberman, 1994). The data validity test in this study was emphasized on data validity testing through data credibility testing, achieved by increased persistence and triangulation techniques. Increased persistence was done by rechecking the data found.

Results

The analysis model used was designed to follow what Bosch and Gascon (2006) call the Reference Epistemological Model (REM). Chevallard stated that the

REM is a theoretical lens for analyzing different mathematical knowledge in institutions, which in this case relates to indefinite integral material (Takeuchi & Shinno, 2020). In this study, REM was used to analyse the content of school mathematics textbooks used as the research data source. The preparation of the REM was based on the integral material contained in the two calculus books used at the University as previously mentioned in the research method. The indefinite integral material is arranged in such a way that it becomes a REM consisting of logos components namely *Theory* (Θ) and *Technology* (θ) and praxis components namely *Techniques* (τ). The REM that is compiled is differentiated on indefinite integral material. The following is a REM that is arranged in relation to the indefinite integral material consisting of two forms of theory, three forms of technology, and five forms of techniques as in Table 3.

Table 3. REM of Indefinite Integral based on Praxeological Organization

Theory		Technology		Techniques	
Θ_1 Conditional ability test	$\Theta_{1.1}$ Presented problems related to derivatives	θ_1 Rules for finding derivatives		τ_1 Operational and algebra	
Θ_2 Indefinite integral	$\Theta_{2.1}$ Presenting the definition of Antiderivative in mathematical form and giving a textual explanation of the mathematical form of Antiderivative and the conditions that must be fulfilled $\Theta_{2.2}$ Presenting a problem with	θ_2 Rules for antiderivatives	$\theta_{2.1}$ Theorem Power Rule (Power rule with powers not equal -1) $\theta_{2.2}$ Theorem Generalized Power Rule $\theta_{2.3}$ Rules for finding antiderivatives of basic trigonome	τ_2 Basic forms of indefinite integrals τ_3 Partial Integral τ_4 Integral of rational functions using partial fractions	$\tau_{2.1}$ Basic substitution technique $\tau_{2.2}$ Rationalizing substitution technique

Theory		Technology		Techniques
	possible antiderivatives of a function so as to obtain the function family of antiderivatives (General Antiderivative)		tric functions	τ_5 Special techniques of integrals on trigonometric function
		θ_3 Indefinite integral is a linear operator	$\theta_{3.1}$ Linearity in the integrand multiplication between constant and function $\theta_{3.2}$ Linearization of the integrand sum operation $\theta_{3.3}$ Linearity in integrant subtraction operations	

Table 3 shows that the theory component has two forms of *theory* namely Θ_1 conditional ability test and Θ_2 indefinite integral with each having its category. The *technology* component consists of three forms, the first form of *technology*, namely θ_1 rules for finding derivatives, the second form of *technology*, namely θ_2 rules for antiderivative, and the third form of *technology*, namely θ_3 indefinite integral linearity. The *technique* component consists of five *techniques*, namely τ_1 operational and algebraic, τ_2 the basic form of indefinite integral, τ_3 partial integral,

τ_4 integral of rational functions using partial fractions, and τ_5 special *technique* of integral on trigonometric functions.

Furthermore, the following table presents the praxeological organization of mathematics textbooks on indefinite integral material as follows.

Table 4. Praxeological Organization of Indefinite Integral Materials in Mathematics Textbook

Praxeology Components	Book			
	B1	B2	B3	B4
Theory $\boldsymbol{\Theta}$				
$\Theta_{1.1}$	V	V	V	–
$\Theta_{2.1}$	V	V	–	–
$\Theta_{2.1}$	–	–	V	V
Technology $\boldsymbol{\theta}$				
θ_1	V	V	V	–
$\theta_{2.1}$	V	V	V	–
$\theta_{2.2}$	–	V	V	–
$\theta_{2.3}$	–	V	V	–
$\theta_{3.1}$	V	V	V	V
$\theta_{3.2}$	V	V	V	V
$\theta_{3.3}$	V	V	V	V
Technique $\boldsymbol{\tau}$				
τ_1	V	V	V	–
$\tau_{2.1}$	–	V	V	–
$\tau_{2.2}$	–	V	–	–
τ_3	–	V	–	–
τ_4	–	–	V	–
τ_5	–	–	–	–
Percentage	56.25	81.25	75.00	25.00

Based on Table 4 above, the percentage of the existence of the praxeology component in each mathematics textbook based on REM is obtained. Book 2 (B2) shows the most conformity compared to other math textbooks which is 81.25%, then B3 is 75.00%, B1 is 56.25%, and B4 is 25.00%. Except for B4, math textbooks present component $\Theta_{1.1}$ in the form of prerequisites of the concept of antiderivative. This becomes very important for students in understanding the definition of antiderivative in components $\Theta_{2.1}$ and $\Theta_{2.2}$.

Furthermore, the percentage of the praxeology component of each book is presented against the REM that has been compiled.

Table 5. Percentage of Praxeological Components of Indefinite Integral Materials against Reference Epistemological Model (REM)

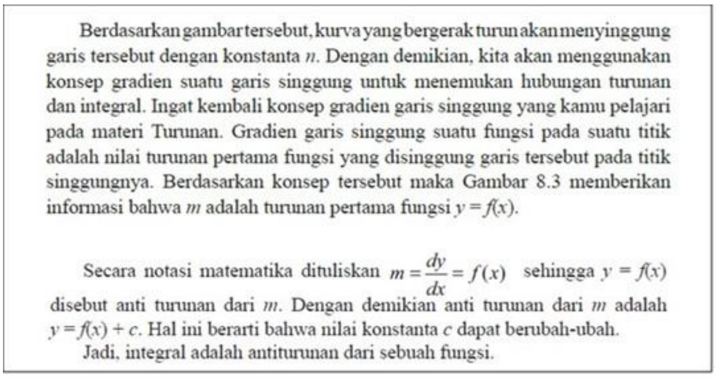
Book	*Theory*			*Technology*			*Technique*		
	On REM	Contain REM	%	On REM	Contain REM	%	On REM	Contain REM	%
B1	$\Theta_{1.1}$	1	33.33	θ_1	1	14.28	τ_1	1	16.67
	$\Theta_{2.1} - \Theta_{2.2}$	2	66.67	$\theta_{2.1} - \theta_{2.3}$	1	14.28	$\tau_{2.1} - \tau_{2.2}$	0	0.00
				$\theta_{3.1} - \theta_{3.3}$	3	71.44	τ_3	0	0.00
							τ_4	0	0.00
							τ_5	0	0.00
B2	$\Theta_{1.1}$	1	33.33	θ_1	1	14.28	τ_1	1	16.67
	$\Theta_{2.1} - \Theta_{2.2}$	1	33.33	$\theta_{2.1} - \theta_{2.3}$	3	42.86	$\tau_{2.1} - \tau_{2.2}$	2	33.33
				$\theta_{3.1} - \theta_{3.3}$	3	42.86	τ_3	1	16.67
							τ_4	0	0.00
							τ_5	0	0.00
B3	$\Theta_{1.1}$	1	33.33	θ_1	1	14.28	τ_1	1	16.67
	$\Theta_{2.1} - \Theta_{2.2}$	1	33.33	$\theta_{2.1} - \theta_{2.3}$	3	42.86	$\tau_{2.1} - \tau_{2.2}$	1	16.67
				$\theta_{3.1} - \theta_{3.3}$	3	42.86	τ_3	0	0.00
							τ_4	1	16.67
							τ_5	0	0.00
B4	$\Theta_{1.1}$	0	0.00	θ_1	0	0.00	τ_1	0	0.00
	$\Theta_{2.1} - \Theta_{2.2}$	1	33.33	$\theta_{2.1} - \theta_{2.3}$	0	0.00	$\tau_{2.1} - \tau_{2.2}$	0	0.00
				$\theta_{3.1} - \theta_{3.3}$	3	42.86	τ_3	0	0.00
							τ_4	0	0.00
							τ_5	0	0.00

Table 5 presents the percentage of each praxeological component in each book against the REM. In the theory component, B1 contains the 1st theory and 2nd theory by 33.33% and 67% respectively, while B2, B3, and B4 have relatively the same percentage of 33.33% for the 1st and 2nd theories, except B4. Furthermore, in the technology component, the completeness of Book 1 has a greater percentage, especially in the 3rd technology compared to B2, B3, and B4. Meanwhile, in the engineering component, B2 has completeness based on REM that is more complete than B1, B3, and B4.

Discussion

The analysis model used in this research was the Reference Epistemology Model (REM) which is based on the opinion of Bosch and Gascon (2006). Chevallard states that REM is a theoretical lens for analyzing different mathematical knowledge in institutions (Takeuchi & Shinno, 2020), which in this case relates to indefinite integral material. Table 3 presents REM related to indefinite integral material, both in the *theory, technology*, and *technique* components. For indefinite integral material, two theories will be taught including Prerequisite Ability and Indefinite Integral. The two components of the *theory*, are further elaborated in-depth in the form of *technology* and *techniques* as presented in Table 3. In teaching indefinite integral material, the first theoretical component that needs to be presented is related to the prerequisite ability test of the derivative concept. Furthermore, it can be further elaborated through technology components related to derivative search rules and techniques related to algebraic operations.

Table 4 presents the percentage of the presence of praxeology components from each book studied based on the REM that has been compiled in Table 3. Overall, B2 has a larger percentage compared to other books, so the presence of praxeology components based on the reference epistemological model can be said to be quite complete, ranging from the components of *theory, technology,* and *technique*. The first theory component presented in the REM relates to the concept of prerequisites for indefinite integral material. Initial problems as prerequisite material in mathematics learning help students link previously acquired understanding or experience with mathematical concepts to be learned. Thus, the initial problem and the explanation of the solution to the initial problem are very important and must be conveyed correctly. If these conditions are not met, it is suspected that it will cause obstacles in student learning so that students become wrong in understanding it. Referring to Table 4 above, except for B4, the mathematics textbooks show the presence of the $\Theta_{1.1}$ *theory* component in the form of prerequisite material. However, in B1 there are things that are suspected to cause learning obstacles to students.



Berdasarkan gambar tersebut, kurva yang bergerak turun akan menyinggung garis tersebut dengan konstanta n. Dengan demikian, kita akan menggunakan konsep gradien suatu garis singgung untuk menemukan hubungan turunan dan integral. Ingat kembali konsep gradien garis singgung yang kamu pelajari pada materi Turunan. Gradien garis singgung suatu fungsi pada suatu titik adalah nilai turunan pertama fungsi yang disinggung garis tersebut pada titik singgungnya. Berdasarkan konsep tersebut maka Gambar 8.3 memberikan informasi bahwa m adalah turunan pertama fungsi $y = f(x)$.

Secara notasi matematika dituliskan $m = \frac{dy}{dx} = f(x)$ sehingga $y = f(x)$ disebut anti turunan dari m. Dengan demikian anti turunan dari m adalah $y = f(x) + c$. Hal ini berarti bahwa nilai konstanta c dapat berubah-ubah.

Jadi, integral adalah antiturunan dari sebuah fungsi.

Figure 2. Initial Material Presentation Related to the Definition of Antiderivatives in Book 1

In the book as Figure 2, it is written that the value of m is the first derivative of the function $y = f(x)$ so that it is denoted $m = dy/dx = f(x)$ so that $y = f(x)$ is the anti-derivative of m. This condition will likely lead to erroneous interpretations for students. This condition will possibly lead to erroneous interpretations for students. The notation $m = dy/dx = f(x)$ shows that the function $f(x)$ is a derived value, thus contradicting the next statement which shows that $y = f(x)$ is the anti-derivative. Based on this, the notation of the value of m should be expressed as $m = dy/dx = f'(x)$. This condition shows that m is the first derivative of the function $y = f(x)$. Nevertheless, the existence of the prerequisite concept will provide a bridge to students regarding the connection between the concepts of derivatives and antiderivatives. Mathematical concepts are interconnected (Confrey, 1981; Novitasari, 2016). The relationship between previously learned concepts and new ones enhances students' understanding, impacting how they learn and their abilities in mathematics (Woolcott, 2013).

The definition of antiderivative as a basic concept in understanding indefinite integrals is important. Orton stated in learning integral concepts, it is important for a student to be able to understand the concept (definition) before the procedure (Tasman et al., 2011). Book 1 (B1) and B2 present textually the theoretical definition of anti-derivative which shows the relationship between the concepts of derivative and antiderivative. On the contrary, in B3, the definition of antiderivative is not textually presented, but in the book a stimulus is given to students to define simply the concept of antiderivative or indefinite integral as in the following Figure 3.

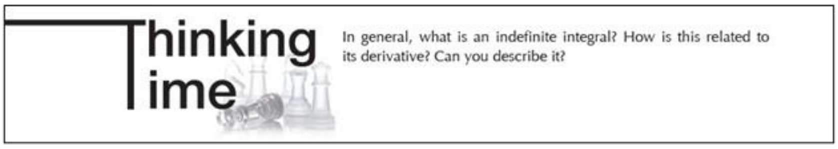


Figure 3. Stimulus definition of Indefinite Integral in Book 3

In mathematics, definitions have an important role. In the learning process, the definition of indefinite integral that is given correctly will form the correct concept of student understanding. Definitions that are presented textually or verbally will provide concept images to students so that students' cognitive structures are formed (Tall & Vinner, 1981). With the definition of indefinite integral as a theory component in praxeology, it is expected to have a positive impact on students' understanding of the integral concept. However, it would be better if students can understand the definition of integral through context or stimulus questions that can bring students to the definition of indefinite integral as presented in Figure 2.

In the *technology* component, B2 and B3 show the overall presence of *technology* as in the REM. Wijayanti and Winslow (2017) stated that the *technology* component is a justification for the *techniques* used in solving a problem or task. In this case, the *technology* component refers to the theorems and properties of indefinite integrals which are then used as justification in solving definite integral problems. In the REM, three forms of *technology* are presented that can be used to justify the *technique*, namely the theorem of the derivative search rule, the theorem of the antiderivative search rule, and the linearity of indefinite integrals. The existence of a complete *technology* component as REM will provide a deep understanding to students so that the techniques used in solving indefinite integrals become correct in the hope that the answers obtained become correct. As in B2, B3 presents the complete *technology* component as REM. Book 3 (B3) even presents the t*echnology* that shows the power rule theorem not only for the value of $n \neq -1$, but also for $n = -1$ which is not found in Book 2 as presented in Figure 4 below.

Since integration is the reverse of differentiation, we have

$$\int \frac{1}{x}\,dx = \ln x + c,\ x > 0, \text{ and}$$

$$\int \frac{1}{ax+b}\,dx = \frac{1}{a}\ln(ax+b) + c,\ ax+b > 0.$$

Figure 4. Technology Component in Book 3

Book 2 (B2) presents more complex *technique* components compared to the other three books. REM presents five technical forms where the second technical form has two pieces. In B2, the fourth *technique* component related to integrating functions using partial fractions and integrating using special forms in trigonometry is not presented. Meanwhile, B3 consists of three *techniques* components, but includes *technique* components that are not presented in other mathematics textbooks, namely integrating using partial fractions.

Referring to Table 5 above, each book shows the percentage of each praxeology component. In the theory component, B1 has a greater percentage than other books. This shows that there is more theoretical content, both the first and second forms that are in accordance with REM. In the *technology* component, B2 and B3 have the same percentage for each component content, while B1 only contains 71.44% and B4 contains 42.86%. As for the *technique* component, B2 has a larger percentage than the other three books. The difference in percentage is based on differences and changes in the curriculum. As found in B1 and B2 used in Indonesia, when viewed as a whole, B2 has a greater percentage of pre-accessology components than B1. This indicates a change in curriculum content. Book 1 (B1) published in 2018 shows less indefinite integral material presented in the book compared to B2.

The indefinite integral search rule theorem can be used as an alternative for students in solving indefinite integral problems. In particular, Radmehr and Drake (2020) stated that most students can solve integral problems by applying basic procedures including theorems on integrals well, although their understanding is still limited. Thus, the presence of the technology component in the textbook, both in B1, B2, B3, and B4 shows that this component has a very crucial role.

The difference in the presentation of the concept of indefinite integral material in each book is based on the learning objectives that have been set while still referring to the learning objectives set by the Government, especially in Indonesia. Changes in the curriculum, including for mathematics subjects, also have an impact on the concepts presented in mathematics including indefinite integral material. Thus, the material components presented in each book are in accordance with the applied curriculum, learning mathematics including indefinite integral material, one of which is directed at character building in accordance with the era of society 5.0 (Daimah & Suparni, 2023).

The urgency of selecting mathematics textbooks is not only based on curriculum changes. The existence of curriculum changes, in the learning process it is also very important to see students' opportunities to do independent learning, one of which is through mathematics textbooks. In addition to opportunities, in the learning process it is also very important to check the curriculum documents (Thompson & Harel, 2021), one of which is the textbook as an integral part of the curriculum. In the process of teaching and learning mathematics, we can examine several different learning opportunities for students, including what textbooks offer and what calculus instructors do in their classes (Hong, 2024). With the results obtained, we hope to be one of the references for stakeholders to be able to make the results of this study as one of the references in the preparation of school mathematics textbooks, especially on the concept of indefinite integrals.

Conclusion

Based on the research results, the compiled Reference Epistemological Model (REM) for indefinite integral material includes two forms of *theory*, three forms of *technology*, and five forms of *technique*. By comparing the textbook content with the compiled REM, the percentage difference in the praxeology components was identified. The praxeology content in textbook B2 is more complex than in the other three textbooks. This complexity is attributed to specific changes in the mathematics curriculum, particularly in the indefinite integral content. This study is limited to three aspects of praxeology—namely *theory, technology*, and *technique*—while the task type aspect has not been analyzed in depth. For this reason, it will be very interesting in future research to analyze mathematics textbooks, especially on indeterminate integral material as a whole to the praxeology component, so that a more complete epistemology model reference and analysis results are produced to be used as a reference in learning in order to improve the achievement of learning outcomes.

References

Amelia, S., & Yadrika, G. (2019). Analisis Kesalahan Siswa SMA dalam Menyelesaikan Soal Integral. *Jurnal Ilmiah Dikdaya*, *9*(1), 124–131. https://doi.org/10.33087/dikdaya.v9i1.132

Aminah, & Kurniawati, K. R. A. (2018). Analisis Kesulitan Siswa Dalam Menyelesaikan Soal Cerita Matematika Topik Pecahan Ditinjau Dari Gender. *JTAM (Jurnal Teori Dan Aplikasi Matematika)*, *2*(2), 118–122. https://doi.org/10.31764/jtam.v2i2.713

Blank, A. A. (1960). Remarks on the teaching of calculus in the secondary school. *The Mathematics Teacher*, *53*(7), 537–539.

Bosch, M., & Gascon, J. (2006). Twenty-Five Years of Didactic Transposition. *ICMI Bulletin*, *58*, 51–65.

Carlson, M., Oehrtman, M., & Engelke, N. (2010). The Precalculus Conecepts Assessment: A Tool for Assesing Students' Reasoning Abilities and Understandings. *Journal Cognition and Instruction*, *28*(2), 113–145. https://doi.org/10.1080/07370001003676587

Cheshier, S. R. (2006). *The Field of Engineering Technology Studying Engineering Technology: A Blueprint for Success*.

Chevallard, Y. (1992). Fundamental concepts in didactics: Perspectives provided by an anthropological approach. In D. R & M. A (Eds.), *Research in Didactique of Mathematics* (pp. 131–167).

Confrey, J. (1981). Concepts, Processes and Mathematics Instruction. *For the Learning of Mathematics*, *2*(1), 8–12.

Daimah, U. S., & Suparni. (2023). Pembelajaran Matematika pada Kurikulum Merdeka dalam Mempersiapkan Peserta Didik di Era Society 5.0. *SEPREN: Journal of Mathematics Education and Applied*, *4*(2), 131–139. https://doi.org/10.36655/sepren.v4i1

Fadillah, A., Firmansyah, M. A., Syarifah, L. L., Rahardjo, S., & Erliani, T. P. (2019). Analisis Learning Obstacle pada Materi Integral. *Imajiner: Jurnal Matematika Dan Pendidikan Matematika*, *1*(6), 243–251. https://doi.org/10.26877/imajiner.v1i6.4802

Hong, D. S. (2024). How do students learn definite integrals? Exploring students' learning opportunities. *School Science and Mathematics*, *124*(1), 32–47. https://doi.org/10.1111/ssm.12605

Hoong, L. Y., Kin, H. W., & Pien, C. L. (2015). Concrete-Pictorial-Abstract: Surveying its origins and charting its future. *The Mathematics Educator*, *16*(1), 1–18. http://math.nie.edu.sg/ame/matheduc/tme/tmeV16_1/TME16_1.pdf

Lisarani, V., Parta, I. N., & Chandra, T. D. (2018). A Comparative Analysis of the Tasks From the Selected Mathematics Textbooks of Singapore and Indonesia. *Jurnal Pendidikan Sains Universitas Negeri Malang*, *6*(4), 94–99.

Mahayukti, G. A., & Dewi, P. K. (2021). Validity and Practicality of Problem-Based Integral Calculus Teaching Materials Assisted with Mathematical Software. In I. G. N. A. Suryaputra, G. Indrawan, & N. P. A. Pratiwi (Eds.), *Proceedings of the 4th International Conference on Innovative Research Across Disciplines (ICIRAD 2021)* (pp. 13–20). Atlantis Press SARL. https://doi.org/10.2991/assehr.k.211222.003

Malihatuddarojah, D., & Prahmana, R. C. I. (2019). Analisis Kesalahan Siswa dalam Menyelesaikan Permasalahan Operasi Bentuk Aljabar. *Jurnal Pendidikan Matematika*, *13*(1), 1–8. https://doi.org/10.22342/jpm.13.1.6668.1-8

Miles, M. B., & Huberman, A. M. (1994). *Qualitative Data Analysis: An Expanded Sourcebook*. Sage Publications Inc.

Muniarti, S., Roza, Y., & Maimunah. (2021). Analisis Keseuaian Materi Himpunan Buku Teks Siswa Matematika Kelas VII terhadap Kurikulum 2013. *Mosharafa: Jurnal Pendidikan Matematika*, *10*(2), 177–188. https://doi.org/10.31980/mosharafa.v10i2.944

Novitasari, D. (2016). Pengaruh Penggunaan Multimedia Interaktif terhadap Kemampuan Pemahaman Konsep Matematis Siswa. *Fibonacci: Jurnal Pendidikan Matematika Dan Matematika*, *2*(2), 8–18. https://doi.org/10.24853/fbc.2.2.8-18

Nurhikmah, S., & Febrian. (2016). Analisis Kesalahan Siswa dalam Menyelesaikan Permasalahan Integral Tak Tentu. *Jurnal Tatsqif*, *14*(2), 218–237. https://doi.org/10.20414/jtq.v14i2.30

Oktaviyanti, R., & Supriani, Y. (2015). Utilizing Microsoft Mathematics in Teaching and Learning Calculus. *Journal on Mathematics Education*, *6*(1), 63–76. https://doi.org/10.22342/jme.6.1.1902.63-76

Orton, A. (1985). When should we teach calculus? *Mathematics in School*, *14*(2), 11–15.

Pramuditya, S. A., Sulaiman, H., & Wahyudin. (2019). Development of instructional media game education on integral and differential calculus. *J. Phys.: Conf. Ser. 1280 042049*. https://doi.org/10.1088/1742-6596/1280/4/042049

Putra, Z. H., & Witri, G. (2017). Anthropological Theory of the Didactic (ATD) A New Research Perspective on Didactic Mathematics in Indonesia. *Jurnal Pigur*, *2*(1), 221–227.

Radmehr, F., & Drake, M. (2020). Exploring Students' Metacognitive Knowledge: The Case of Integral Calculus. *Education Sciences*, *10*(3), 55. https://doi.org/10.3390/educsci10030055

Rahmi, A., Hairi, A. P., Lukman, A., & Nasution, A. (2020). Analisis Kemampuan Bernalar Siswa SMA dalam Memaknai Permasalahan Integral Berbasis Konteks. *Fibonacci: Jurnal Pendidikan Matematika Dan Matematika*, *1*(2), 1–9. https://doi.org/10.24114/jfi.v1i2.21910

Ramda, A. H., Kurnila, V. S., & Jundu, R. (2018). Implementasi Kurikulum 2013 pada Standar Isi Buku Teks Matematika Kelas VIII. *Jurnal Eksakta Pendidikan (JEP)*, *2*(2), 162–169. https://doi.org/10.24036/jep/vol2-iss2/231

Ramelan, M., & Wijaya, A. (2019). A Comparative Analysis of Indonesian and Singaporean Mathematics Textbooks from the Perspective of Mathematical Creativity: A Case Statistics and Probability. *Journal of Physics: Conference Series*, *1320*(1), 012037. https://doi.org/10.1088/1742-6596/1320/1/012037

Rizqi, M. M., Wijayanti, D., & Basir, M. A. (2021). Analisis Buku Teks Matematika Materi Himpunan Menggunakan Model Prakseologi. *Delta: Jurnal Ilmiah Pendidikan Matematika*, *9*(1), 57–76. https://doi.org/10.31941/delta.v9i1.1226

Robinson, W. B. (1970). The effects of two semesters of secondary school calculus on students' first and second quarter calculus grades at the University of Utah. *Journal for Research in Mathematics Education*, *1*(1), 57–60.

Takeuchi, H., & Shinno, Y. (2020). Comparing the Lower Secondary Textbooks of Japan and England: a Praxeological Analysis of Symmetry and Transformations in Geometry. *International Journal of Science and Mathematics Education*, *18*(2), 791–810. https://doi.org/10.1007/s10763-019-09982-3

Tall, D. (1997). Functions and Calculus. In A. J. Bishop, K. Clements, C. Keitel, & J. Kilpatrick (Eds.), *International Handbook of Mathematics Education* (pp. 289–325). Kluwer.

Tall, D., & Vinner, S. (1981). Concept Image and Concept Definition in Mathematics with Particular Reference to Limits and Continuity. *Educational Studies in Matematics*, *12*, 151–169.

Tanujaya, B., Prahmana, R. C. I., & Mumu, J. (2017). Mathematics instruction, problems, challenges and opportunities: A case study in Manokwari Regency, Indonesia. *World Transactions on Engineering and Technology Education*, *15*(3), 287–291.

Tasman, F., Hertog, J. Den, Zulkardi, Z., & Hartono, Y. (2011). Helping Students Acquainted with Multiplication in Rectangular Model. *Journal on Mathematics Education*, *2*(2), 185–198. https://doi.org/10.22342/jme.2.2.747.185-198

Thompson, P. W., & Harel, G. (2021). Ideas foundational to calculus learning and their links to students' difficulties. *ZDM – Mathematics Education*, *53*(3), 507–519. https://doi.org/10.1007/s11858-021-01270-1

Usman, Hasbi, M., Bambang, R. M., & Mahathir. (2019). Deskripsi Kinerja dan Eksplorasi Kesulitan Belajar Siswa SMA dalam Menyelesaikan Masalah Kalkulus. *Numeracy*, *6*(2), 218–228. https://doi.org/10.46244/numeracy.v6i2.478

Wijaya, A. (2019). The Relationships between Indonesian Fourth Graders' Difficulties in Fractions and the Opportunity to Learn Fractions: A Snapshot of TIMSS Results. *International Journal of Instruction*, *10*(4), 221–236. https://doi.org/10.12973/iji.2017.10413a

Wijayanti, D., & Winslow, C. (2017). Mathematical practice in texbooks analysis: Praxeological refence models, the case of proportion. *REDIMAT*, *6*(3), 307–330. https://doi.org/10.17583/redimat.2017.2078

Woolcott, G. (2013). Mathematics Networks and Curriculum Concepts. In V. Steinle, L. Ball, & C. Bardini (Eds.), *Procceedings of the 36th annual conference of the Mathematics Education Research Group of Australasia*. MERGA.

Yang, D.-C., & Sianturi, I. A. (2017). An Analysis of Singaporean versus Indonesian Textbooks Based on Trigonometry Content. *EURASIA Journal of Mathematics, Science and Technology Education*, *13*(7), 3829–3848. https://doi.org/10.12973/eurasia.2017.00760a

Yuniarti, S., Hariyati Nur Indah Sari, T., & Khotimah, H. (2018). Utilizing GeoGebra in Application of Integral Module for High School Students in Balikpapan, East Borneo Indonesia. In Haviluddin, A. P. Wibawa, A. Pranolo, P. P. Widagdo, H. J. Setyadi, & M. Taruk (Eds.), *Proceedings of the 2017 International Conference on Education and Technology (2017 ICEduTech)* (Vol. 144, pp. 252–256). Atlantis Press. https://doi.org/10.2991/icedutech-17.2018.50

Zakaria, E., & Salleh, T. S. (2015). Using Technology in Learning Integral Calculus. *Mediterranean Journal of Social Sciences*, *6*(5), 144–148. https://doi.org/10.5901/mjss.2015.v6n5s1p144

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